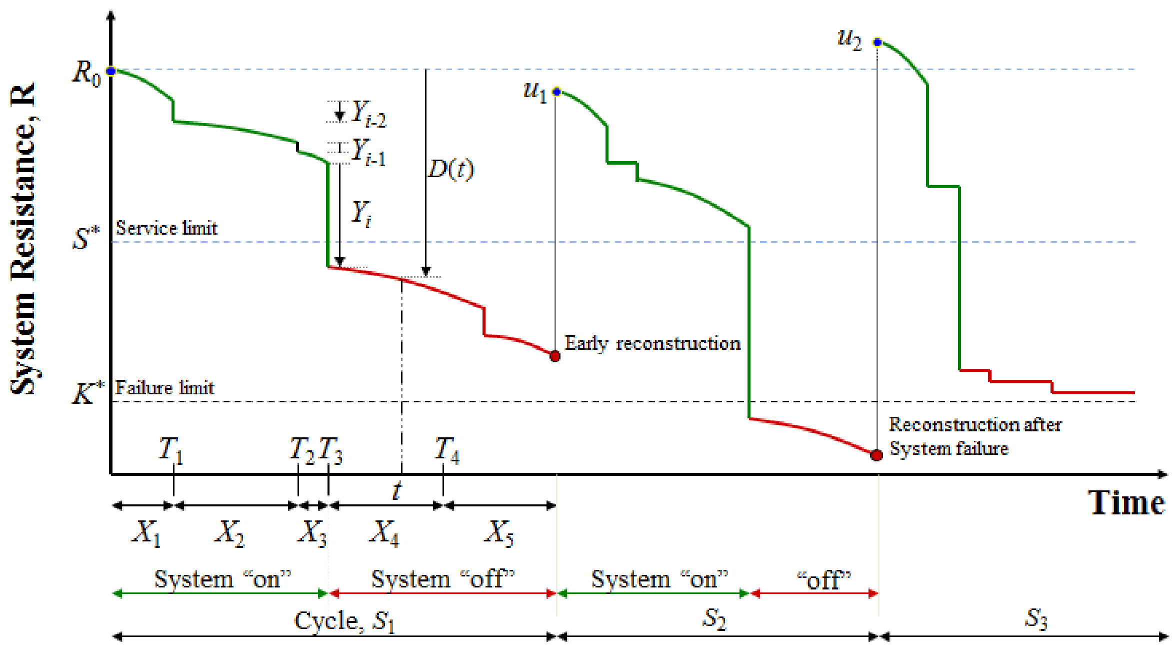
AN INTRODUCTION TO LIFE CYCLE ANALYSIS

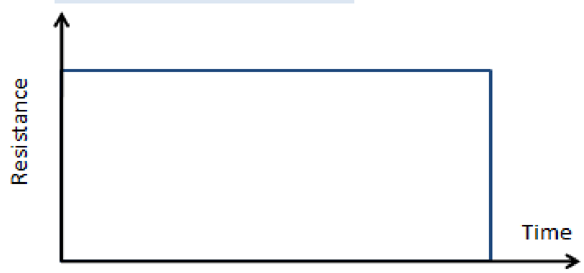
MECHANICAL BEHAVIOUR OF THE SYSTEM

Generic System Life Cycle

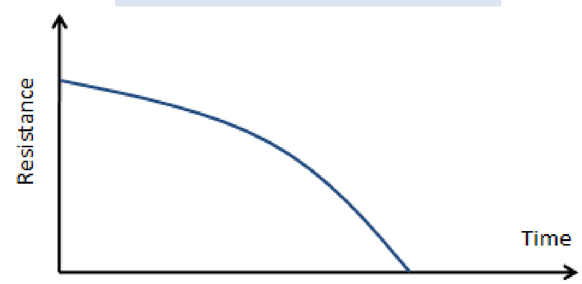


POSSIBLE BEHAVIOUR MODELS – WITHOUT SYSTEM RECONSTRUCTION

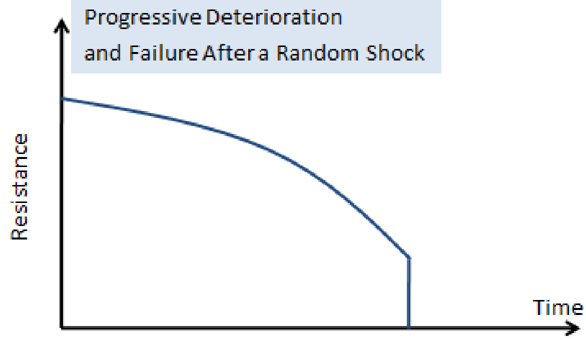
Abandon After First Failure



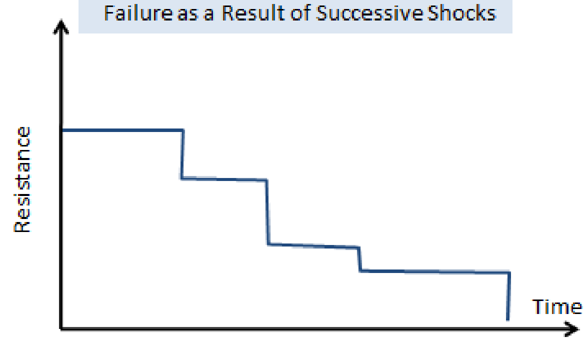
Progressive Deterioration to Failure



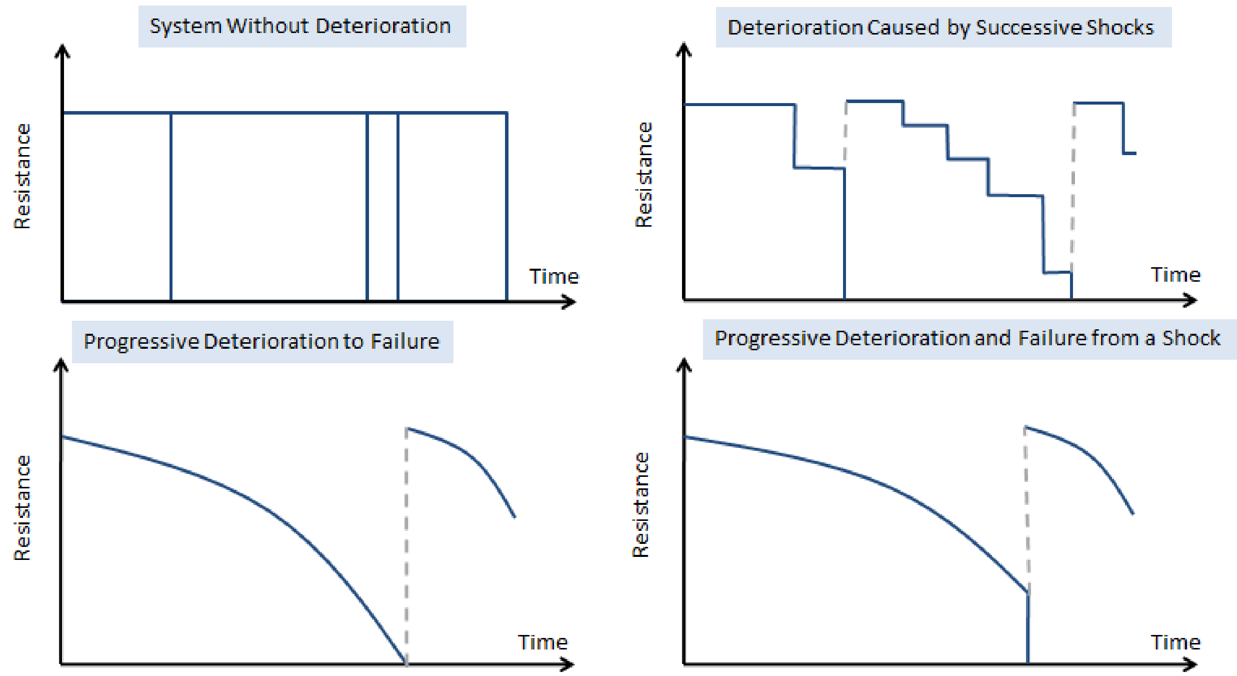
Progressive Deterioration and Failure After a Random Shock



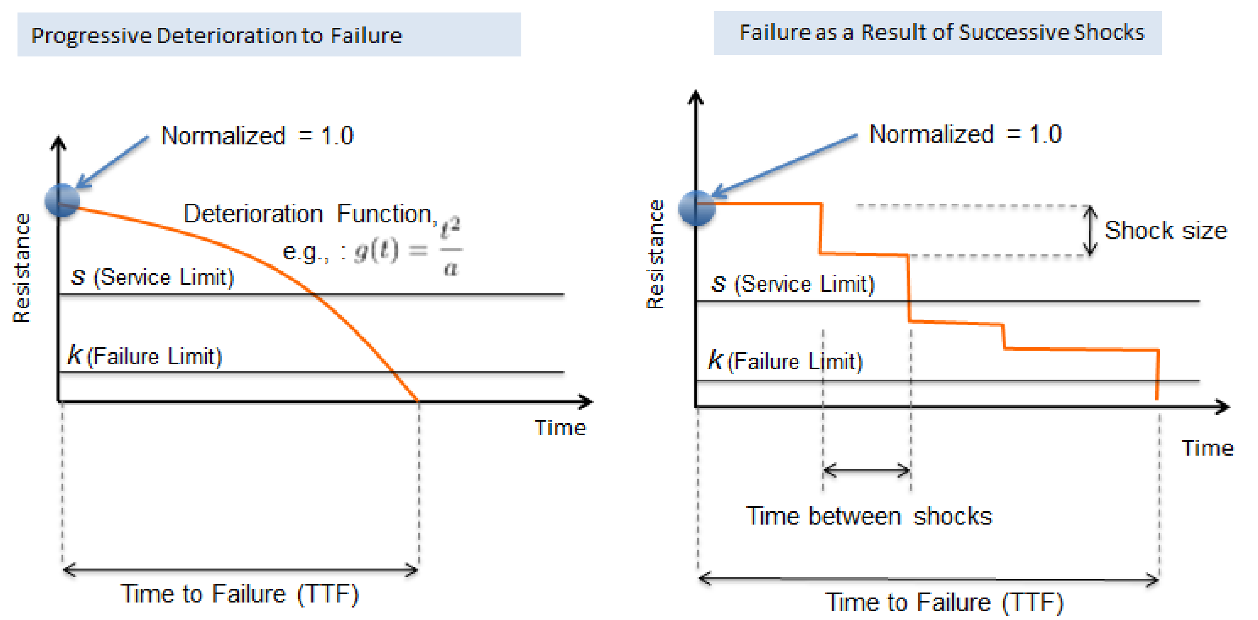
Failure as a Result of Successive Shocks



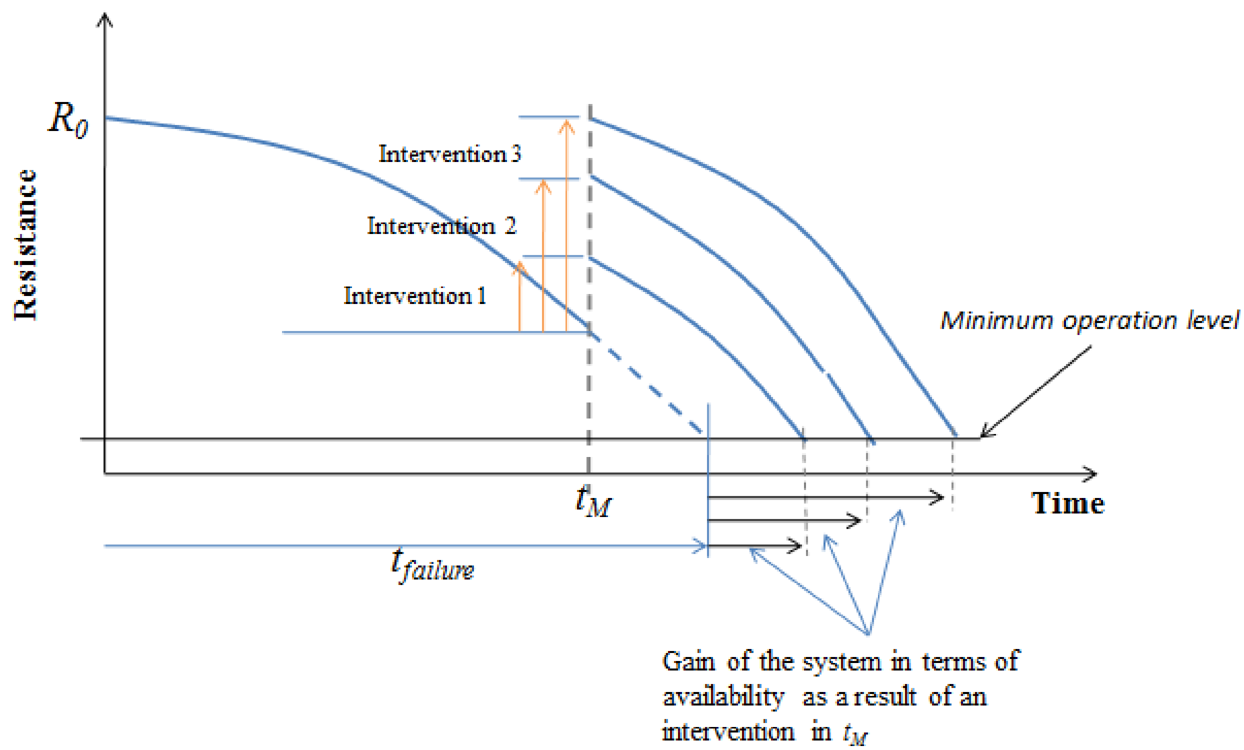
POSSIBLE BEHAVIOUR MODELS – WITH SYSTEM RECONSTRUCTION



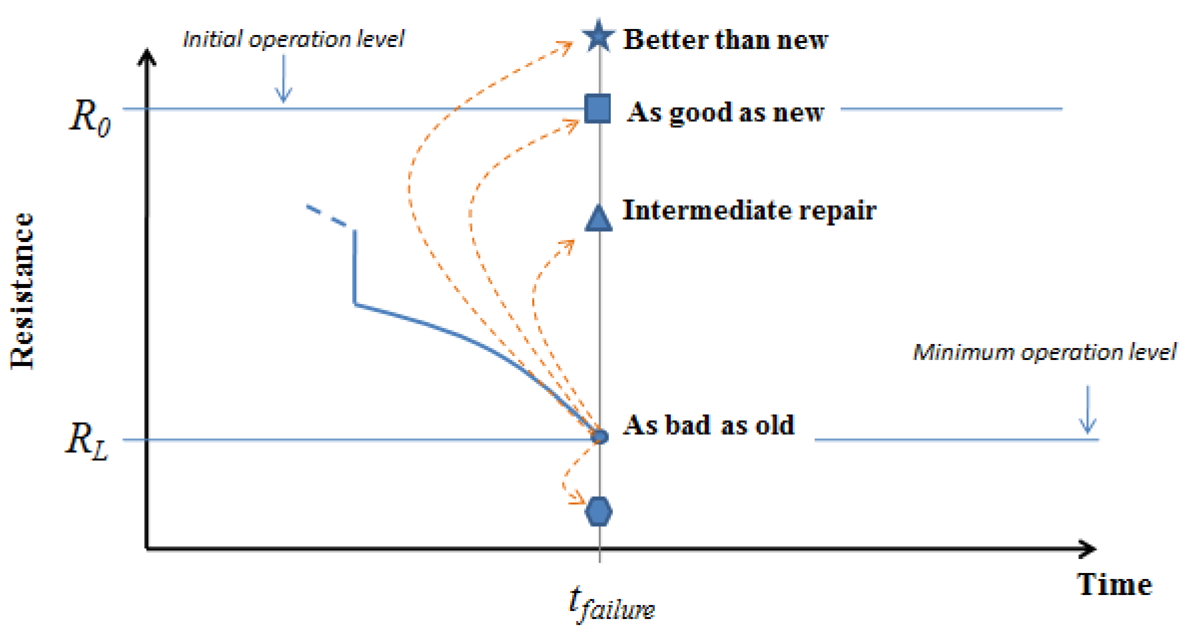
2R LCA TERMINOLOGY EXPLAINED



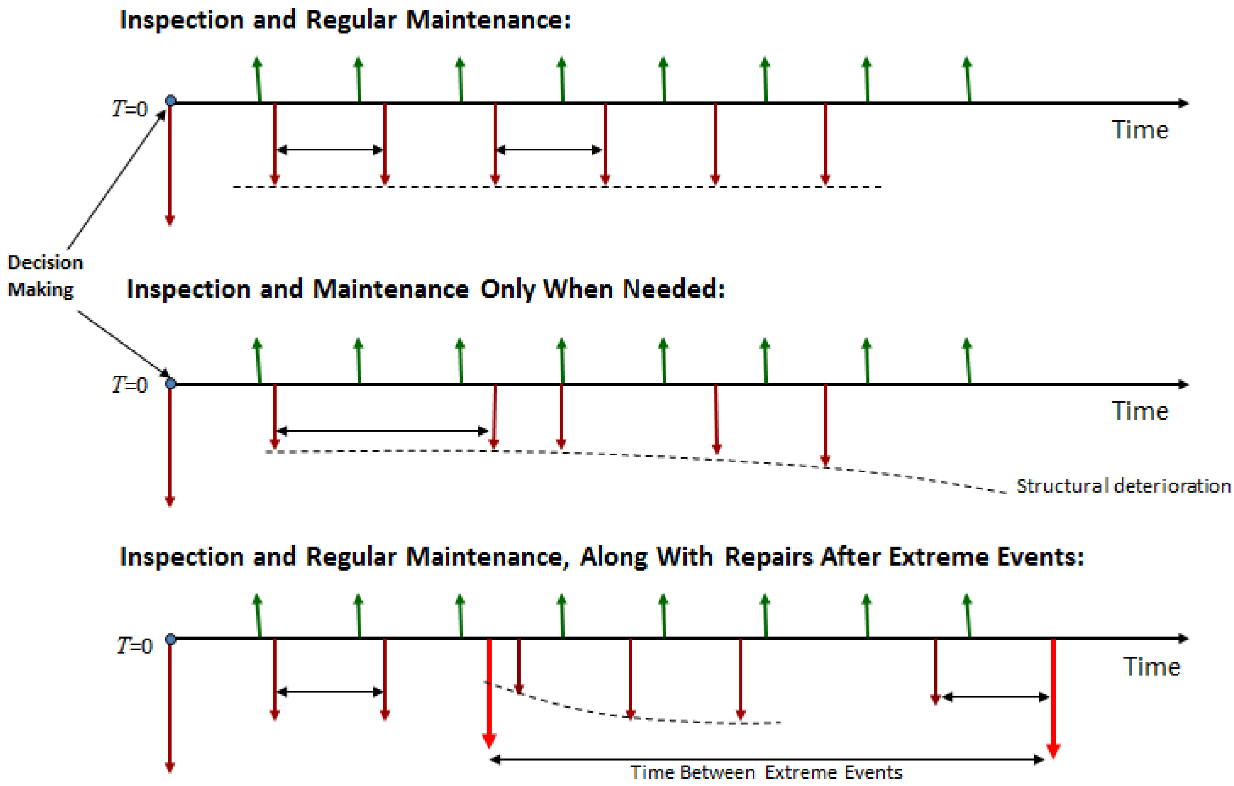
EFFECT OF AN INTERVENTION ON THE SYSTEM'S AVAILABILITY



RECONSTRUCTION APPROACHES



MAINTENANCE SCHEMES



COST ANALYSIS

FINANCIAL VIABILITY

The financial analysis must be carried out for *t*=0, which entails the calculation of the project's **Net Present Value**:

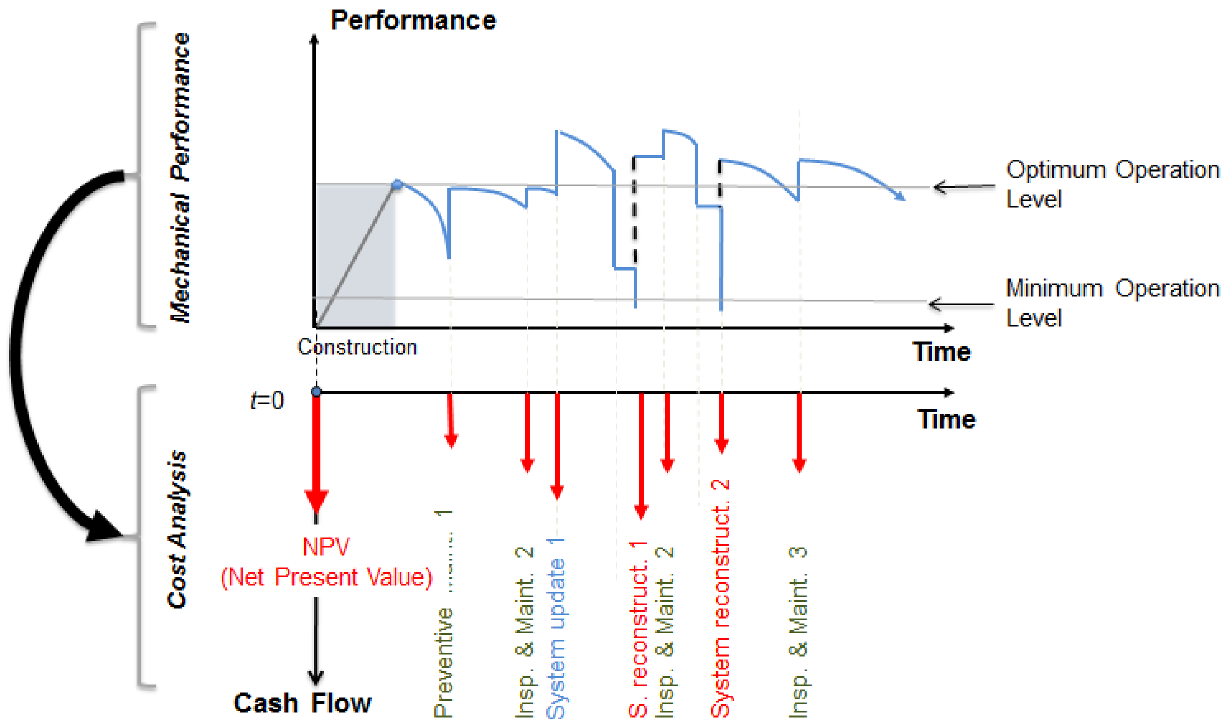
$$VPN = \sum_{i=1}^{n} C(t_i) \cdot e^{-\gamma \cdot t_i}$$

The discount rate must take two factors into account:

1. **Pure Time Value of Money (δ):** the interest rate demanded by an investor for postponing his consumption and making available capital to a borrower.
2. **Inflation Premium (ρ):** if the lender anticipates inflation during the term of loan he demands an extra return to compensate for the loss in purchase power of money.

$$e^{-\gamma(t)t} = e^{-\delta t} \cdot e^{-\rho t} = e^{-(\rho+\delta)t}$$

PERFORMANCE AND COST ANALYSIS CORRESPONDANCE



UTILITY FUNCTION

Utility Function: $$Z(\mathbf{R}) = B(\mathbf{R},t) - C(\mathbf{R},t) - D(\mathbf{R},t)$$

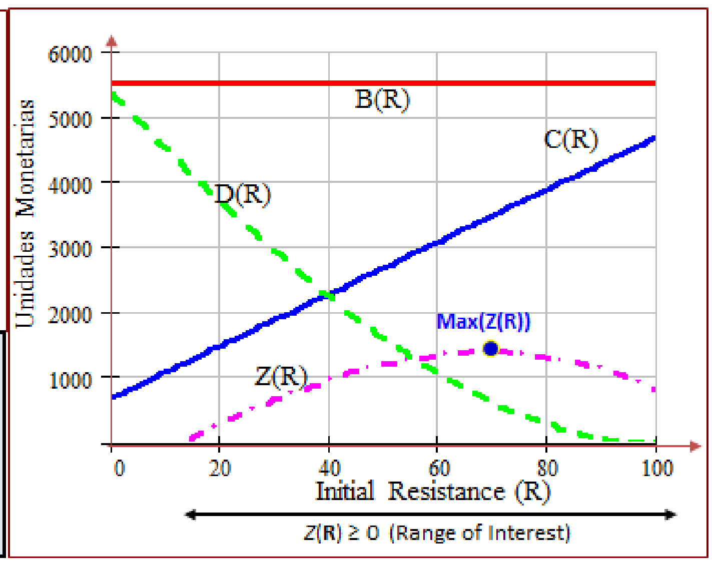
B(**R**, *t*): Benefit provided by the system

C(**R**, *t*): Cost associated with the system's deployment

D(**R**, *t*): Expected cost associated with the operation of the system

Optimum Design Objective:

Max(Z(R))



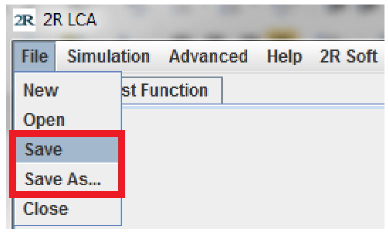
FILE OPERATIONS

2R LCA is capable of reading and writing **.2rl** files, which contain the complete description of a specific life cycle model (approach, deterioration model, analysis parameters, cost function, etc).

These files are the means for 2R LCA users to save their work, as well as the medium of distribution of models that could be of interest to other 2R LCA users.

SAVING MODELS

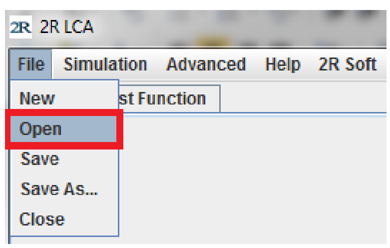
Even if a model isn't yet complete, a user can decide to save its information for later use. In order to do this, the user must navigate through the **File** menu and select the **Save** or **Save As…** option:



The difference between **Save** and **Save As…** is that, while **Save** will only ask for the file's name and destination once and will then overwrite that same file on any subsequent uses, **Save As…** will ask for the file's name and destination every time it is invoked. Thus, **Save As…** is to be used whenever a user wants to save modifications made to a file without modifying the base file.

OPENING MODELS

In order to load the information contained inside a **.2rl** file, the user must navigate through the **File** menu and select the **Open** option:

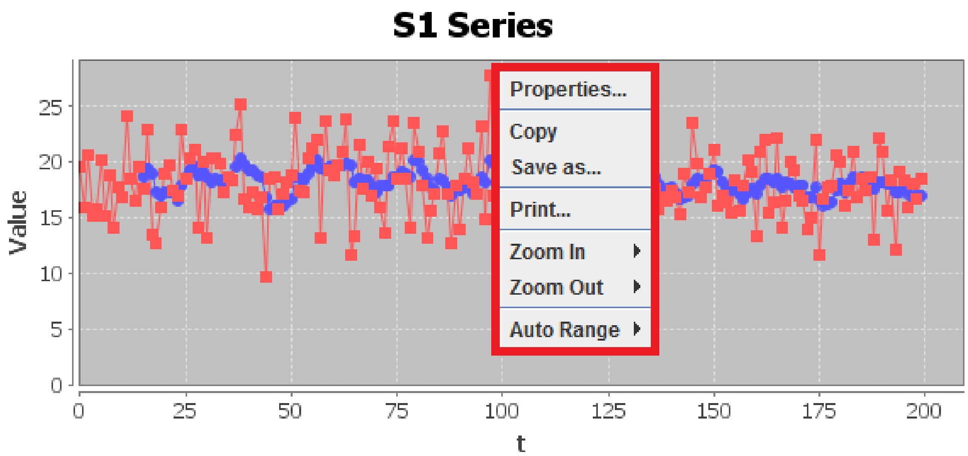


If no errors occur, the loaded model is shown in the **Main** and **Cost Function** tabs (approach, deterioration model, analysis parameters, cost function, etc).

BASICS

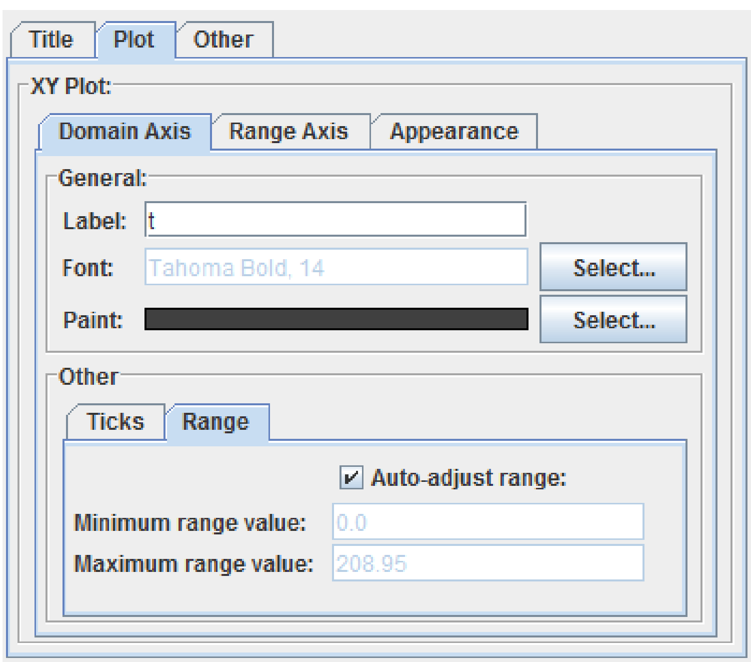
GRAPHS

All of the graphs generated in 2R Soft provide a wide array of options in the form of a context menu. **The context menu appears when you right-click over a graph:**



PROPERTIES PANE

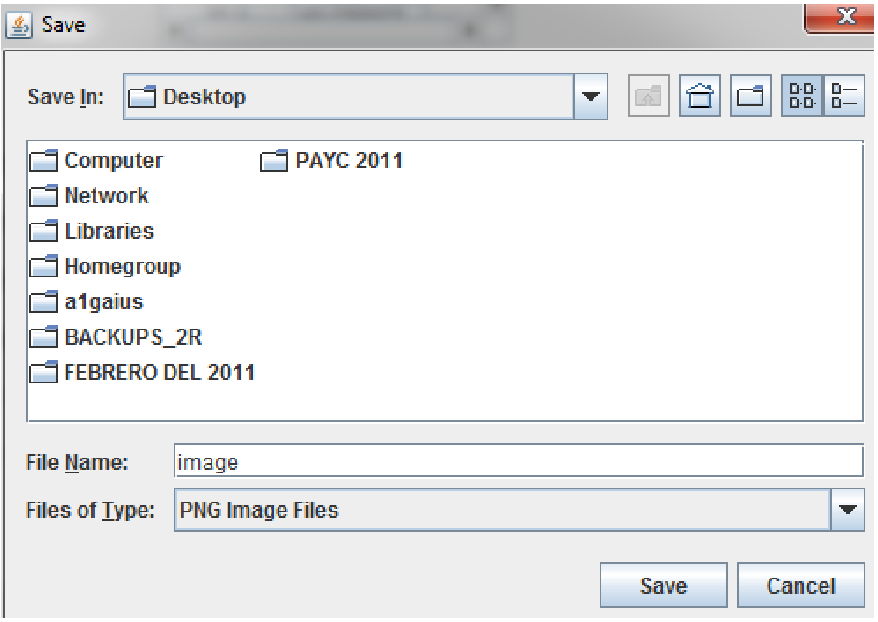
If you select the **Properties…** option, a properties pane appears. The properties pane lets you change the graph title, axis names, axis ranges, and font size.



COPY AND SAVE AS…

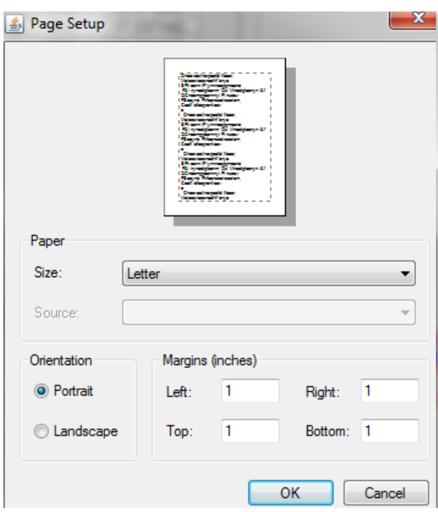
If you select **COPY**, the graph is copied to the system clipboard, so you can **PASTE** it anywhere else (Microsoft Word, Microsoft PowerPoint, etc).

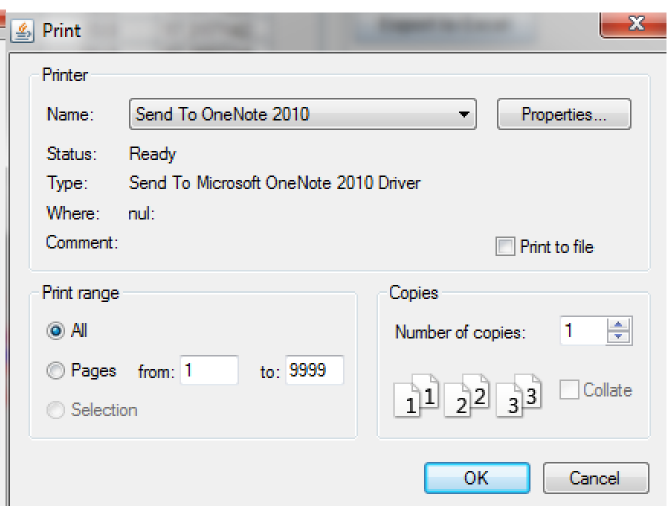
Meanwhile, if **Save As…** is selected, 2R Soft will save the graph as a **PNG** image file in your hard disk after selecting the desired output folder and file name:



PRINT

The **Print** option does just that: it sends the graph to the printer of your choice (local or networked):



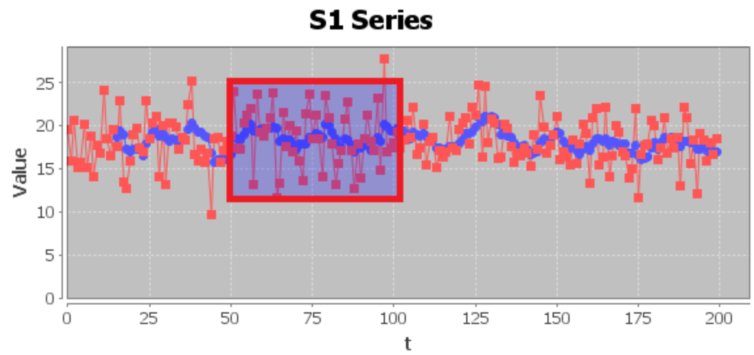


SCALE OPTIONS

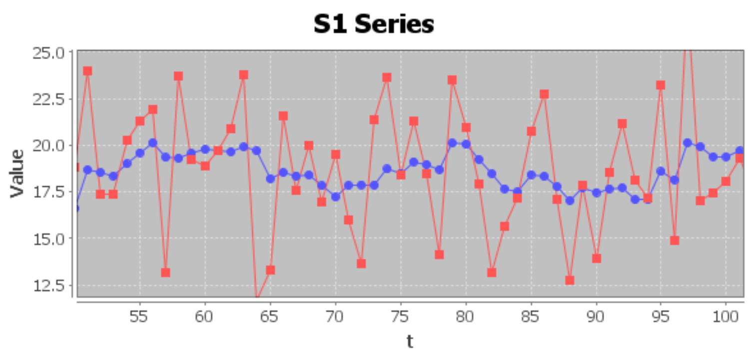
The **Auto Range**, **Zoom In**, and **Zoom Out** options are a quick way to inspect the graph. If a very specific range is needed for an axis, we highly recommend the Properties Pane.

MANUAL ZOOM IN

For user convenience, all **2R Soft graphs support manual zoom in by regions.** If you're interested in a specific region**, hold your left-click and drag the mouse** to generate a highlighted box around that region:



The end result:



EQUATION EDITOR

User-entered equations are common in 2R Soft. This section of the document explains the use of the equation editor.

FUNCTIONS AND OPERATIONS

Equations can contain the following functions and operations:

Function/Op.	Description	Usage (A and B are declared variables or numeric values)
+	Addition.	A+B Example: 4+7=11
-	Subtraction.	A-B Example: 4-7=-3
*****	Multiplication.	A*B Example: 4*7=28
/	Division.	A/B Example: 6/4=1.5
^	Returns the value of the first operand raised to the power of the second operand. (Microsystems)	A^B Example: 5^3=125 Example: 5^(-3)=1/125
%	Modulo operation. Divides the value of one expression by the value of another, and returns the remainder. (MSDN)	A%B Example: 7%3=1 Example: 67%10=7
cos	Returns the trigonometric cosine of an angle. (Microsystems)	cos(A), where A is in radians Example: cos(3.14) ≈ 1.0

sin	Returns the trigonometric sine of an angle. (Microsystems)	sin(A), where A is in radians Example: sin(3.14) ≈ 0.0
tan	Returns the trigonometric tangent of an angle. (Microsystems)	tan(A), where A is in radians Example: tan(3.14) ≈ 0.0
acos	Returns the arc cosine of an angle, in the range of 0.0 through pi. (Microsystems)	acos(A), where A is the value whose arc cosine is to be returned. Example: acos(1)=0.0
asin	Returns the arc sine of an angle, in the range of -pi/2 through pi/2 (Microsystems)	asin (A), where A is the value whose arc sine is to be returned. Example: asin(0)=0.0
atan	Returns the arc tangent of an angle, in the range of -pi/2 through pi/2. (Microsystems)	atan(A), where A is the value whose arc tangent is to be returned. Example: atan(0)=0.0
sqrt	Returns the correctly rounded positive square root of a positive real number. (Microsystems)	sqrt(A), where A is a real positive number. Example: sqrt(9)=3
sqr	Returns the value of the argument squared (to the power of 2).	sqr(A) Example: sqr(-4)=16 Example: sqr(2)=4
ln	Returns the natural logarithm (base e) of a real value. (Microsystems)	ln(A), where A is a positive real number greater than zero. Example: ln(1)=0 Example: ln(e^2)=2
min	Returns the smaller of two real values. That is, the result is the value closer to negative infinity. (Microsystems)	min(A,B) Example: min(4,9)=4 Example: min(-4,-11)=-11
max	Returns the greater of two real values. That is, the result is the argument closer to positive infinity. If the arguments have the same value, the result is that same value. (Microsystems)	max(A,B) Example: max(4,9)=9 Example: max(-4,-11)=-4
ceil	Returns the smallest (closest to negative infinity) real value that is not less than the argument and is equal to a mathematical integer. (Microsystems)	ceil(A) Example: ceil(9.23)=10 Example: ceil(-1.25)=-1
floor	Returns the largest (closest to positive infinity) value that is not greater than the argument and is equal to a mathematical integer (Microsystems)	floor(A) Example: floor(9.23)=9 Example: floor(-1.25)=-2
abs	Returns the absolute value of a real value. If the argument is not negative, the argument is returned. If the argument is negative, the negation of the argument is returned. (Microsystems)	abs(A) Example: abs(4.15)=4.15 Example: abs(-4.3)=4.3 Example: abs(0)=0
neg	Changes the sign of the value received as an argument (negative to positive or positive to negative).	neg(A) Example: neg(-1)=1 Example: neg(1)=-1 Example: neg(0)=0
rnd	Returns a pseudo-random real value between 0.0 (included) and the value received as an argument (excluded).	rnd(A) Example: rnd(50)=0,1.45,2.78,49.9 Example: rnd(-20)=0,-1.45,-18.392,-19.61
exp	Returns Euler's number e raised to the power of a real value. (Microsystems)	exp(A) Example: exp(0)=1 Example: exp(1)=e Example: exp(-2)=1/(e^2)
log	Returns the logarithm (base 10) of a real value. (Microsystems)	log(A), where A is a positive real number greater than zero. Example: log(1)=0 Example: log(10^2)=2

PROBABILITY DISTRIBUTION TYPES

When declaring a variable with a known distribution, the user can select one of many types of probability distributions.

Distribution	Description	Parameters
Beta	The *beta* distribution has shape parameters $\alpha > 0$ and $\beta > 0$ over the interval (a, b), where $a < b$. It has density: $f(x) = (x - a)^{\alpha-1}(b - x)^{\beta-1}/[B(\alpha, \beta)(b - a)^{\alpha+\beta-1}]$ for $a < x < b$, and 0 elsewhere. It has the following distribution function: $F(x) = I_{\alpha,\beta}(x) = \int_a^x(\xi - a)^{\alpha-1}(b - \xi)^{\beta-1}/[B(\alpha, \beta)(b - a)^{\alpha+\beta-1}]d\xi$, for $a < x < b$ (Simard)	Alpha – shape parameter, alpha > 0 Beta – shape parameter, beta > 0 a – lower bound of the interval b – upper bound of the interval, b > a
Binomial	The binomial distribution with parameters n and p, where n is a positive integer and $0 <= p <= 1$. Its mass function is given by: $p(x) = \text{nCr}(n, x)p^x(1 - p)^{n-x} = n!/[x!(n - x)!]\ p^x(1 - p)^{n-x}$ for $x = 0, 1, 2,\ldots n$, and its distribution function is: $F(x) = \sum_{j=0}^{x}\text{nCr}(n, j)\ p^j(1 - p)^{n-j}$ for $x = 0, 1, 2,\ldots n$, where nCr(n, x) is the number of possible combinations of x elements chosen among a set of n elements. (Simard)	p – probability of success on each trial (0 ≤p≤1) n – number of trials (integer), n > 0
Chi Square	The *chi-square* distribution with n degrees of freedom, where n is a positive integer. Its density is: $f(x) = x^{(n/2)-1}e^{-x/2}/(2^{n/2}\Gamma(n/2))$, for $x > 0$ where $\Gamma(x)$ is the gamma function. The *chi-square* distribution is a special case of the *gamma* distribution with shape parameter $n/2$ and scale parameter1/2. (Simard)	n – degrees of freedom (integer), n>0
Deterministic	Distribution that represents a constant value, *val*. Consequently: $f(x) = \begin{cases}1\ if\ x = val\\0\ if\ x \neq val\end{cases}$, $F(x) = \begin{cases}1\ if\ x \geq val\\0\ if\ x < val\end{cases}$	Value – any real number
Discrete Uniform	The *discrete uniform* distribution over the integers in the range [i, j]. Its mass function is given by: $p(x) = 1/(j - i + 1)$ for $x = i, i + 1,\ldots, j$ and 0 elsewhere. The distribution function is: $F(x) = (\text{floor}(x) - i + 1)/(j - i + 1)$ for $i <= x <= j$ and its inverse is: $F^{-1}(u) = i + (j - i + 1)u$ for $0 <= u <= 1$. (Simard)	Min. – lower bound (integer) Max. – upper bound (integer) (Max. > Min.)
Exponential	The *exponential* distribution with mean $1/\lambda$ where $\lambda > 0$. Its density is: $f(x) = \lambda e^{-\lambda x}$ for $x >= 0$, its distribution function is: $F(x) = 1 - e^{-\lambda x}$, for $x >= 0$, and its inverse distribution function is: $F^{-1}(u) = - \ln(1 - u)/\lambda$, for $0 < u < 1$ (Simard)	Lambda – rate parameter, lambda > 0
F-Distribution	The Fisher F distribution with n and m degrees of freedom, where n and m are positive integers. Its density is: $f(x) = \Gamma((n + m)/2)n^{n/2}m^{m/2}/[\Gamma(n/2)\Gamma(m/2)]x^{(n-2)/2}/(m + nx)^{(n+m)/2}$, for $x > 0$. where $\Gamma(x)$ is the gamma function (Simard)	D.O.F. 1 – the n degrees of freedom (integer), D.O.F. 1 > 0 D.O.F. 2 – the m degrees of freedom (integer), D.O.F. 2 > 0

Gamma	**The *gamma* distribution with shape parameter $\alpha > 0$ and scale parameter $\lambda > 0$. The density is:** **$f(x) = \lambda^{\alpha} x^{\alpha-1} e^{-\lambda x}/\Gamma(\alpha)$, for $x > 0$,** **where Γ is the gamma function, defined by:** **$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx$.** **In particular, $\Gamma(n) = (n - 1)!$ when n is a positive integer.** **(Simard)**	**Alpha – shape parameter, alpha > 0** **Lambda – scale parameter, lambda>0**
Geometric	The *geometric* distribution with parameter p, where $0 < p < 1$. Its mass function is: $p(x) = p\,(1 - p)^x$, for $x = 0, 1, 2,\ldots$ The distribution function is given by: $F(x) = 1 - (1 - p)^{x+1}$, for $x = 0, 1, 2,\ldots$ and its inverse is: $F^{-1}(u) = \text{floor}(\ln(1 - u)/\ln(1 - p))$, for $0 <= u < 1$ (Simard)	p – probability of success on each trial (0 < p < 1)
Gumbel	The Gumbel distribution, with location parameter δ and scale parameter $\beta \neq 0$. Using the notation $z = (x - \delta)/\beta$, it has density: $f(x) = e^{-z} e^{-e^{-z}}/\vert\beta\vert$, for $-\infty < x < \infty$. and distribution function: $F(x) = e^{-e^{-z}}$, for $\beta > 0$ $F(x) = 1 - e^{-e^{-z}}$, for $\beta < 0$ (Simard)	Beta – scale parameter, beta ≠ 0 Delta – location parameter, any real number
Hypergeometric	The *hypergeometric* distribution with k elements chosen among l, m being of one type, and $l - m$ of the other. The parameters m, k and l are positive integers where $1 <= m <= l$ and $1 <= k <= l$. Its mass function is given by: $p(x) = \text{nCr}(m, x)\text{nCr}(l - m, k - x)/\text{nCr}(l, k)$ for $\max(0, k - l + m) <= x <= \min(k, m)$ where $\text{nCr}(n, x)$ is the number of possible combinations of x elements chosen among a set of n elements. (Simard)	m – number of elements of one type (integer), m > 0 l – total elements (integer), l > 0 k – number of elements chosen among l (integer), k > 0
Logistic	The *logistic* distribution. It has location parameter α and scale parameter $\lambda > 0$. The density is: $f(x) = (\lambda e^{-\lambda(x-\alpha)})/((1 + e^{-\lambda(x-\alpha)})^2)$ for $-\infty < x < \infty$. and the distribution function is: $F(x) = 1/[1 + e^{-\lambda(x-\alpha)}]$ for $-\infty < x < \infty$. For $\lambda = 1$ and $\alpha = 0$, one can write: $F(x) = (1+\tanh(x/2))/2$. The inverse distribution function is given by: $F^{-1}(u) = \ln(u/(1 - u))/\lambda + \alpha$ for $0 <= u < 1$ (Simard)	Alpha – location parameter, any real number Lambda – scale parameter, lambda>0
Lognormal	The *lognormal* distribution. It has scale parameter μ and shape parameter $\sigma > 0$. The density is: $f(x) = ((2\pi)^{1/2}\sigma x)^{-1} e^{-(\ln(x)-\mu)2/(2\sigma 2)}$ for $x > 0$, and 0 elsewhere. The distribution function is: $F(x) = \Phi((\ln(x)-\mu)/\sigma)$ for $x > 0$, where Φ is the standard normal distribution function. Its inverse is given by: $F^{-1}(u) = e^{\mu+\sigma\Phi\text{-}1(u)}$ for $0 <= u < 1$ If $\ln(Y)$ has a *normal* distribution, then Y has a *lognormal* distribution with the same parameters. (Simard)	log mu – scale parameter, any real number log sigma – shape parameter, log sigma > 0

Negative Binomial

The negative binomial distribution with real parameters γ and p, where $\gamma > 0$ and $0 <= p <= 1$. Its mass function is:
$p(x) = \Gamma(\gamma + x)/(x!\, \Gamma(\gamma))p^{\gamma}(1 - p)^{x}$, for $x = 0, 1, 2,\ldots$
where Γ is the gamma function.

If γ is an integer, $p(x)$ can be interpreted as the probability of having x failures before the γ-th success in a sequence of independent Bernoulli trials with probability of success p.

(Simard)

Gamma – number of failures until the experiment is stopped, Gamma > 0
p – success probability in each experiment, $0 \leq p \leq 1$

Normal

The *normal* distribution. It has mean μ and variance σ^2. Its density function is:
$f(x) = e^{-(x-\mu)2/(2\sigma 2)}/((2\pi)^{1/2}\sigma)$ for $-\infty < x < \infty$, where $\sigma > 0$.

When $\mu = 0$ and $\sigma = 1$, we have the *standard normal* distribution, with corresponding distribution function:
$F(x) = \Phi(x) = \int_{-\infty}^{x} e^{-t2/2}\, dt/(2\pi)^{1/2}$ for $-\infty < x < \infty$.

(Simard)

Mean – self-explanatory, any real number
Standard Deviation – self-explanatory, Std. Dev. > 0

Pareto

The *Pareto* family, with shape parameter $\alpha > 0$ and location parameter $\beta > 0$. The density for this type of Pareto distribution is:
$f(x) = \alpha\beta^{\alpha}/x^{\alpha+1}$ for $x >= \beta$, and 0 otherwise.

The distribution function is:
$F(x) = 1 - (\beta/x)^{\alpha}$ for $x >= \beta$,

and the inverse distribution function is:
$F^{-1}(u) = \beta(1 - u)^{-1/\alpha}$ for $0 <= u < 1$

(Simard)

Alpha – shape parameter, alpha > 0
Beta – location parameter, beta > 0

Poisson

The *Poisson* distribution with mean $\lambda >= 0$. The mass function is:
$p(x) = e^{-\lambda}\lambda^{x}/(x!)$, for $x = 0, 1,\ldots$
and the distribution function is:
$F(x) = e^{-\lambda}\sum_{j=0}^{x} \lambda^{j}/(j!)$, for $x = 0, 1,\ldots.$

(Simard)

Lambda – mean, lambda ≥ 0

Student's T

The *Student-t* distribution with n degrees of freedom, where n is a positive integer. Its density is:
$f(x) = [\Gamma((n + 1)/2)/(\Gamma(n/2)(\pi n)^{1/2})][1 + x^2/n]^{-(n+1)/2}$ for $-\infty < x < \infty$,
where $\Gamma(x)$ is the gamma function

(Simard)

D.O.F – degrees of freedom (integer), D.O.F > 0

Triangular

The *triangular* distribution with domain $[a, b]$ and mode (or shape parameter) m, where $a <= m <= b$. The density function is:

$f(x) =$	$2(x - a)/[(b - a)(m - a)]$	for $a <= x <= m$,
$f(x) =$	$2(b - x)/[(b - a)(b - m)]$	for $m <= x <= b$,
$f(x) =$	0	elsewhere,

the distribution function is:

$F(x) =$	0	for $x < a$,
$F(x) =$	$(x - a)^2/[(b - a)(m - a)]$	if $a <= x <= m$,
$F(x) =$	$1 - (b - x)^2/[(b - a)(b - m)]$	if $m <= x <= b$,
$F(x) =$	1	for $x > b$,

and the inverse distribution function is given by:

$F^{-1}(u) =$	$a + ((b - a)(m - a)u)^{1/2}$	if $0 <= u <= (m - a)/(b - a)$,
$F^{-1}(u) =$	$b - ((b - a)(b - m)(1 - u))^{1/2}$	if $(m - a)/(b - a) <= u <= 1$

(Simard)

a – lower bound of the domain, any real number
b – upper bound of the domain, any real number
mode – shape parameter, any real number

"$a \leq mode \leq b$" must be satisfied

Uniform

The *uniform* distribution over the interval $[a, b]$. Its density is:
$f(x) = 1/(b - a)$ for $a <= x <= b$, and 0 elsewhere.

The distribution function is:
$F(x) = (x - a)/(b - a)$ for $a <= x <= b$

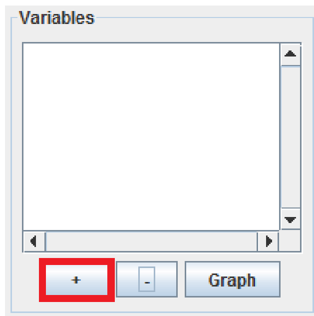
and its inverse is:
$F^{-1}(u) = a + (b - a)u$ for $0 <= u <= 1$ (Simard)

Min. – lower bound
Max. – upper bound
(Max. > Min.)

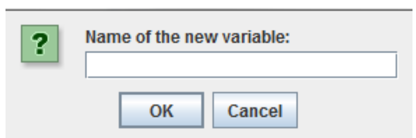
Weibull	The *Weibull* distribution with shape parameter $\alpha > 0$, location parameter δ, and scale parameter $\lambda > 0$. The density function is: $f(x) = \alpha\lambda^{\alpha}(x - \delta)^{\alpha-1}e^{-(\lambda(x-\delta))\alpha}$ for $x > \delta$. the distribution function is: $F(x) = 1 - e^{-(\lambda(x-\delta))\alpha}$ for $x > \delta$, and the inverse distribution function is: $F^{-1}(u) = (-\ln(1 - u))^{1/\alpha}/\lambda + \delta$ for $0 <= u < 1$ (Simard)	Alpha – shape parameter, alpha > 0 Lambda – scale parameter, lambda>0 Delta – location parameter, any real number

ADDING VARIABLES

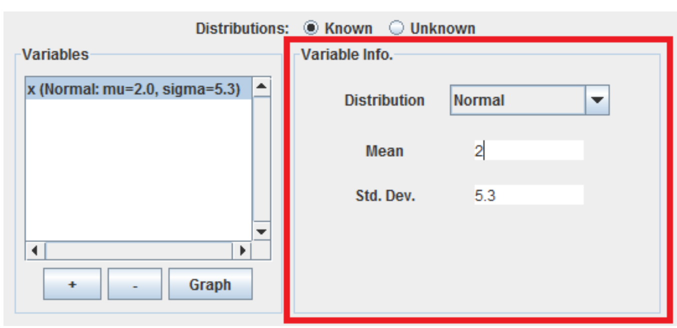
In order to add a variable to a model, the user must left-click over the **+** button that can be found in the **Main** tab:



The user will be asked to enter the new variable's name:

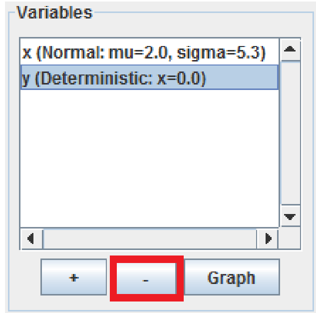


As long as there is no other previously declared variable with the same name as the one entered, the new variable will show up in the list of variables. Afterwards, the user must change the variable's information in the **Variable Info** pane, depending on the type of distribution that is to be assigned to the variable:



REMOVING VARIABLES

Removing variables is straight-forward. The user just needs to select the variable that is to be removed from the simulation model in the **Variables** pane and then left-click over the **–** button:

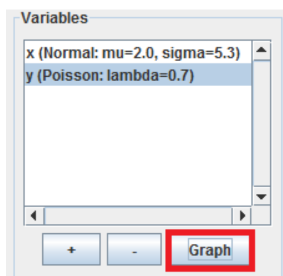


In the screenshot above, the variable **y** would disappear from the model after clicking the **–** button.

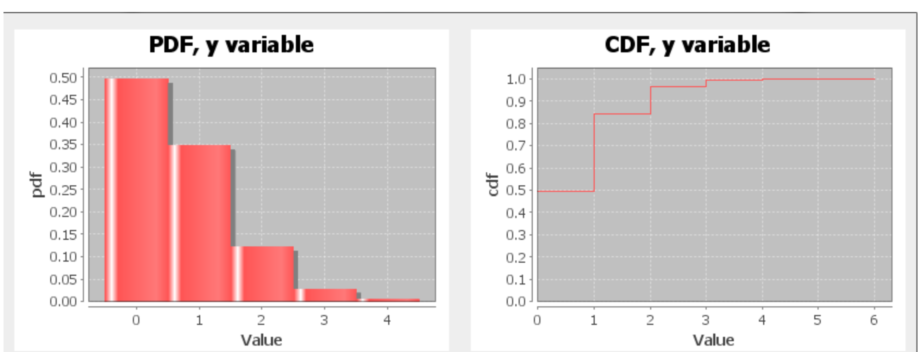
GRAPHING A SINGLE VARIABLE

In order to verify that the parameters of the variables coincide with what the user expects, 2R Soft provides the option to visualize the PDF (probability density function) and CDF (cumulative density function) curves associated with each variable before running a model.

To view the behavior of a particular variable in graph form, the user must select such variable in the **Variables** pane and then left-click over the **Graph** button:

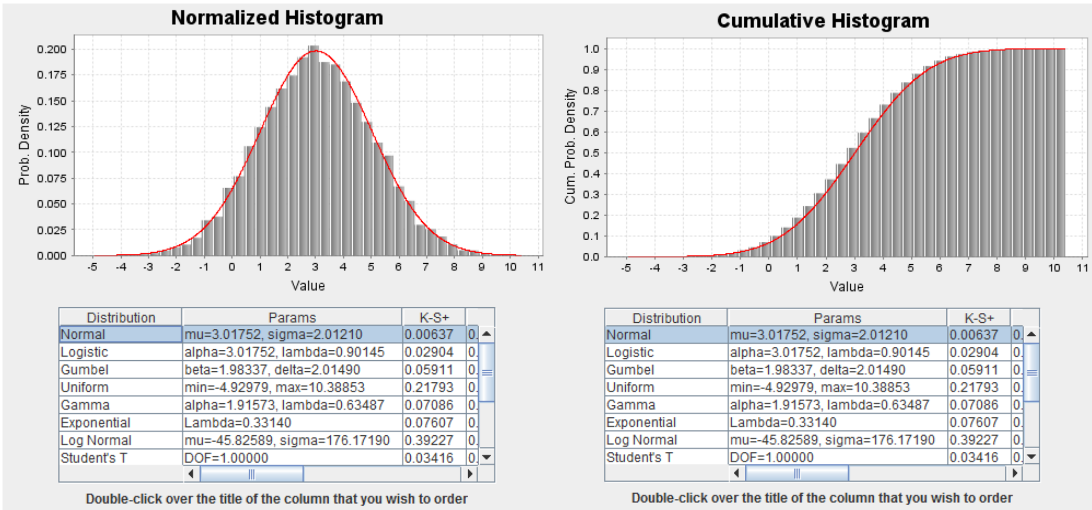


A new window will pop-up with the PDF and CDF curves:



NORMALIZED AND CUMULATIVE HISTOGRAMS

Normalized and Cumulative histograms are found all throughout 2R Soft. These two types of graphs are of great importance, considering that they show the stochastic tendencies of a data set. With them, a goodness-of-fit table is displayed with various probability distributions and the best estimates for their parameters, along with different goodness-of-fit tests:



The distribution selected in the goodness-of-fit table is juxtaposed with the histograms (red curve). Refer to the Probability Distribution Types section for more information on the various distribution types supported by 2R Soft.

GOODNESS-OF-FIT (GOF) STATISTICS

To decide whether to accept or reject a proposed probability distribution for the generated data, it is necessary to at least use one goodness-of-fit statistic as a criterion.

Col.	Test Type	Explanation	Critical Values
A-D	Anderson-Darling	The Anderson-Darling test is defined as: H_0: The data follow a specified distribution. H_a: The data do not follow the specified distribution Test Statistic: The Anderson-Darling test statistic is defined as $A^2 = -N - S$ where $S = \sum_{i=1}^{N} \frac{(2i-1)}{N} [\ln F(Y_i) + \ln(1 - F(Y_{N+1-i}))]$ ***F*** is the cumulative distribution function of the specified distribution. Note that the Y_i are the *ordered* data. The test is a one-sided test and the hypothesis that the distribution is of a specific form is rejected if the test statistic, A, is greater than the critical value. (SEMATECH2)	The critical values for the Anderson-Darling test are dependent on the specific distribution that is being tested. (SEMATECH2) For Normal and Lognormal distributions: **alpha**: 0.1, 0.05, 0.025, 0.01 **A^2_{crit}**: 0.631, 0.752, 0.873, 1.035 For Weibull and Gumbel distributions: **alpha**: 0.1, 0.05, 0.025, 0.01 **A^2_{crit}**: 0.637, 0.757, 0.877, 1.038 (Annis)

K-S | **Kolmogorov-Smirnov**

The Kolmogorov-Smirnov test is defined by:

H_0:	The data follow a specified distribution
H_a:	The data do not follow the specified distribution
Test Statistic:	The Kolmogorov-Smirnov test statistic is defined as

$$D = \max_{1 \le i \le N} \left(F(Y_i) - \frac{i-1}{N}, \frac{i}{N} - F(Y_i) \right)$$

where ***F*** is the theoretical cumulative distribution of the distribution being tested which must be a continuous distribution (i.e., no discrete distributions such as the binomial or Poisson), and it must be fully specified.

(SEMATECH1)

An attractive feature of this test is that the distribution of the K-S test statistic itself does not depend on the underlying cumulative distribution function being tested. Therefore, the critical values are universal. (SEMATECH1)

For samples with more than 35 values: (ERI)

Significance	0.20	0.15	0.10	0.05	0.01
Critical Value	$\frac{1.07}{\sqrt{N}}$	$\frac{1.14}{\sqrt{N}}$	$\frac{1.22}{\sqrt{N}}$	$\frac{1.36}{\sqrt{N}}$	$\frac{1.63}{\sqrt{N}}$

K-S+ and K-S- | Kolmogorov-Smirnov+ and Kolmogorov-Smirnov-

Given a sample of *n* independent uniforms U_i over [0, 1], the *Kolmogorov-Smirnov+* statistic D_n^+ and the *Kolmogorov-Smirnov-* statistic D_n^-, are defined by

$D_n^+ = \max_{1 <= j <= n}(j/n - U_{(j)})$,
$D_n^- = \max_{1 <= j <= n}(U_{(j)} - (j - 1)/n)$,

where the $U_{(j)}$ are the U_i sorted in increasing order. Both statistics follows the same distribution function, i.e. $F_n(x) = P[D_n^+ <= x] = P[D_n^- <= x]$

(Simard)

The same from the K-S test.

CVM | Cramér-von Mises

Given a sample of *n* independent uniforms U_i over [0, 1], the Cramér-von Mises statistic W_n^2 is defined by
$W_n^2 = 1/12n + \sum_{j=1}^{n}(U_{(j)} - (j\text{-}0.5)/n)^2$,
where the $U_{(j)}$ are the U_i sorted in increasing order. The distribution function (the cumulative probabilities) is defined as $F_n(x) = P[W_n^2 <= x]$

(Simard)

As with the K-S test, the critical values are universal:

	Significance				
N	**0.20**	**0.15**	**0.10**	**0.05**	**0.01**
2	0.138	0.149	0.162	0.175	0.186
10	0.125	0.142	0.167	0.212	0.32
20	0.128	0.146	0.172	0.217	0.33
30	0.128	0.146	0.172	0.218	0.33
60	0.128	0.147	0.173	0.220	0.33
100	0.129	0.147	0.173	0.220	0.34

(ReliaSoftCorp)

WG | Watson G

Given a sample of *n* independent uniforms U_i over [0, 1], the *G* statistic is defined by

$G_n = (n)^{1/2}\max_{1 <= j <= n}[j/n - U_{(j)} + \text{bar}(U)_n - 1/2]$
$= (n)^{1/2}(D_n^+ + \text{bar}(U)_n - 1/2)$,

where the $U_{(j)}$ are the U_i sorted in increasing order, $\text{bar}(U)_n$ is the average of the observations U_i, and D_n^+ is the Kolmogorov-Smirnov+ statistic. The distribution function (the cumulative probabilities) is defined as $F_n(x) = P[G_n <= x]$

(Simard)

From 2R Soft WG CDF code:

	Significance				
N	**0.20**	**0.15**	**0.10**	**0.05**	**0.01**
2	0.609	0.636	0.670	0.718	0.787
10	0.692	0.728	0.773	0.843	0.979
20	0.711	0.747	0.794	0.866	1.010
30	0.719	0.755	0.802	0.875	1.021
60	0.728	0.765	0.813	0.887	1.034
100	0.734	0.770	0.818	0.893	1.041
1000	0.745	0.782	0.831	0.906	1.055
10000	0.749	0.786	0.834	0.909	1.059

WU | Watson U

Given a sample of *n* independent uniforms u_i over [0, 1], the Watson statistic U_n^2 is defined by
$W_n^2 = 1/12n + \sum_{j=1}^{n}[u_{(j)} - (j\text{-}0.5)/n]^2$,
$U_n^2 = W_n^2 - n(\text{bar}(u)_n - 1/2)^2$.
where the $u_{(j)}$ are the u_i sorted in increasing order, and $\text{bar}(u)_n$ is the average of the observations u_i. The distribution function (the cumulative probabilities) is defined as $F_n(x) = P[U_n^2 <= x]$

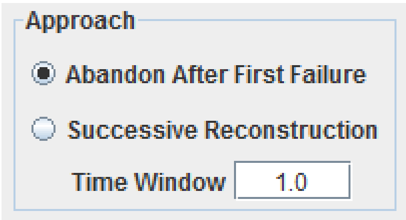
(Simard)

From 2R Soft WU CDF code:

	Significance				
N	**0.20**	**0.15**	**0.10**	**0.05**	**0.01**
2	0.122	0.132	0.143	0.154	0.164
10	0.116	0.130	0.150	0.183	0.255
20	0.116	0.131	0.151	0.185	0.262
30	0.117	0.131	0.151	0.185	0.264
60	0.117	0.131	0.151	0.186	0.266
100	0.117	0.131	0.152	0.186	0.267
1000	0.117	0.131	0.152	0.187	0.268
10000	0.117	0.131	0.152	0.187	0.268

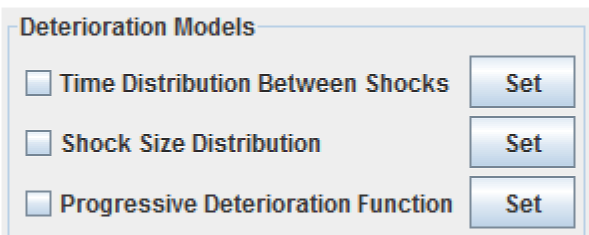
INPUT DATA

APPROACH



- **Abandon After First Failure:** a simulation is considered to be complete when the system fails for the first time.
- **Successive Reconstruction:** a simulation is only considered to be complete when the time window (t=time window) is reached. Therefore, if the system fails before the time window is reached, it is reconstructed in accordance with the "**R After Reconstruction**" settings.

DETERIORATION MODELS



A life cycle analysis model is correct if it fits in one of four cases:

- **Deterioration Function only.**
- **Time Distribution Between Shocks only.**
- **Shock Size Distribution with Time Distribution Between Shocks.**
- **Deterioration Function with Time Distribution Between Shocks and Shock Size Distribution.**

You can activate/deactivate the different attributes of the model by checking/unchecking the boxes next to them. In the screenshot above, all attributes are disabled, since all of the boxes are unchecked.

TIME DISTRIBUTION BETWEEN SHOCKS

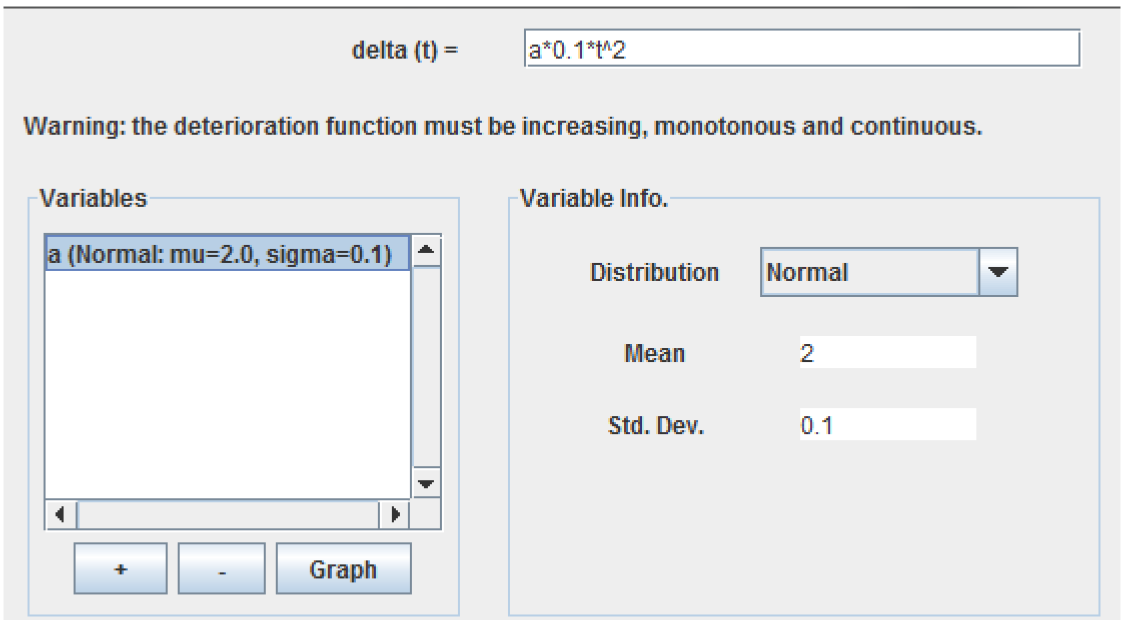
The time distribution between shocks is a random variable that defines the time lapse between one shock on the system and the next. Therefore, **this input is necessary if shocks are to be taken into account throughout the life cycle analysis simulations.** Refer to the Probability Distribution Types section for more information on the various distribution types supported by 2R LCA.

SHOCK SIZE DISTRIBUTION

The shock size distribution is a random variable that **describes the amount of damage that the system receives during a shock**. If the shock size distribution is disabled (left with the box unchecked), every shock leads to the failure of the system. Refer to the Probability Distribution Types section for more information on the various distribution types supported by 2R LCA.

DETERIORATION FUNCTION

The deterioration function is a **monotonous, increasing, and continuous function of time**. When declaring this function, the variable "t" must be used to represent time. Other random variables can be used in the deterioration function and should be declared in the "variables" list. An example of a deterioration function would be:



Note that the variable "t" doesn't have to be declared in the "variables" list.

Refer to the Equation Editor section for information on supported functions and equation syntax in general.

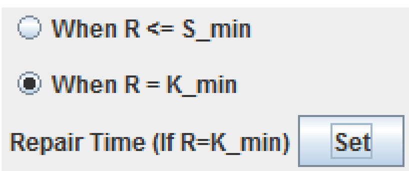
ANALYSIS OPTIONS

LIMITS

A life cycle analysis model can have two different resistance limits:

- **K_min**: this limit indicates the resistance at which a system failure takes place. Hence, the system's resistance is always greater or equal to K_min.
- **S_min**: this limit indicates the resistance from which system maintenance is suggested. S_min must be greater or equal to K_min, and affects the maintenance behavior as detailed in the following section.

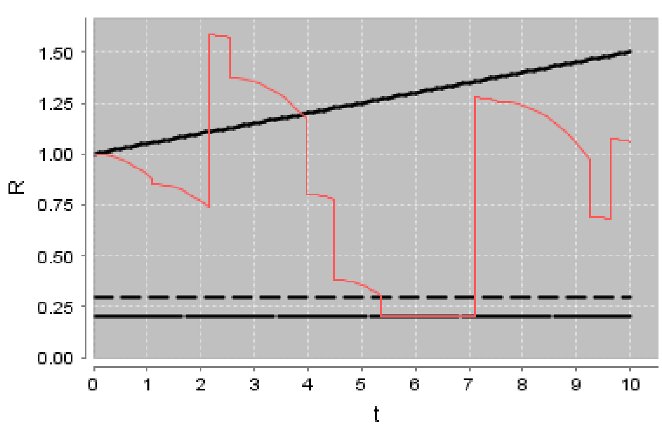
RECONSTRUCT



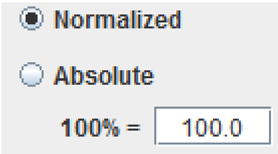
Reconstruction can be done under two profiles:

- **When R <= S_min:** if this setting is activated, the system will be reconstructed whenever its resistance falls on or below S_min. Given that S_min is greater or equal to K_min, this behavior can lead to system reconstruction before failure.
- **When R = K_min:** this setting only allows system reconstruction when system failure is reached. Therefore, the S_min value is completely ignored under this mode.

The **Repair Time (If R=K_min)** is a random variable that represents the down time of the system if failure occurs. As such, it has a direct effect on the system's availability. Refer to the Probability Distribution Types section for more information on the various distribution types supported by 2R LCA. In the image below, the repair time is shown between t=5 and t=7:



SCALE OPTIONS



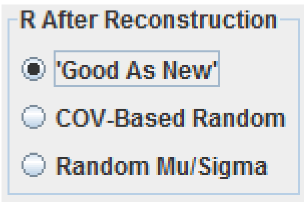
System scale greatly impacts the entire LCA model and relates to the range of values for system resistance (R). This setting must be taken into account when declaring the **Shock Size Distribution** and the **Deterioration Function**. For example, shocks of size 100 may be reasonable in an absolute scale of 100%=1000, but make no sense in a normalized scale (100%=1).

- **Normalized:** the normalized scale uses a range from 0.0 to 1.0 for system resistance (R).
- **Absolute:** absolute scales can use an arbitrary range of resistance values. The “100%=” textbox receives the value at which the system’s resistance is considered to be 100% (R=100%).

R AFTER RECONSTRUCTION

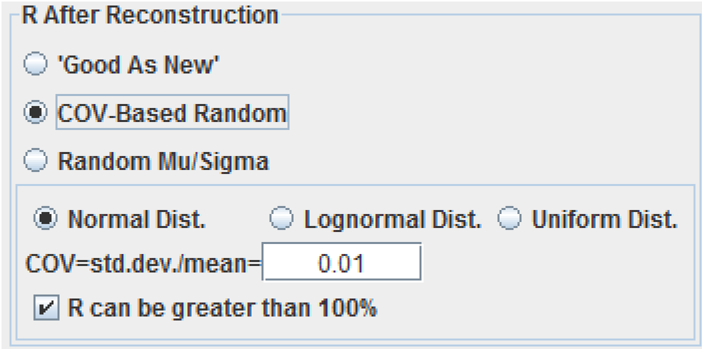
The system’s resistance after being reconstructed shouldn’t necessarily be 100%. Some might argue that the reconstructed system should have a resistance lower than 100% when the system isn’t replaced with a new one, while others might argue that the reconstructed system should have a resistance greater than 100% because of advancements in technology and expertise as time passes by.

‘GOOD AS NEW’



This is the default setting. Under these conditions, the system is reconstructed to exactly R=100% every time it undergoes maintenance.

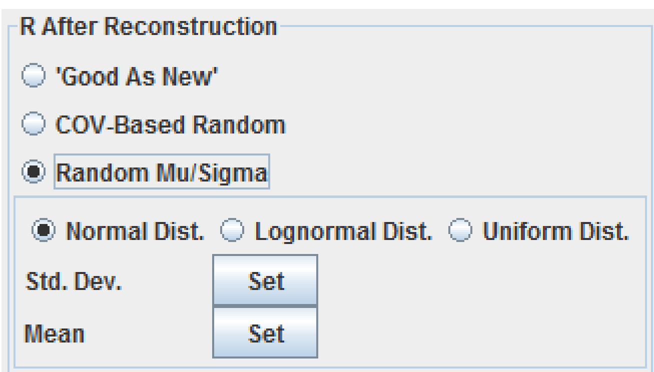
COV-BASED RANDOM



The COV-Based Random setting generates basic randomness in the value of system resistance (R) after maintenance. There are three important parameters under these conditions:

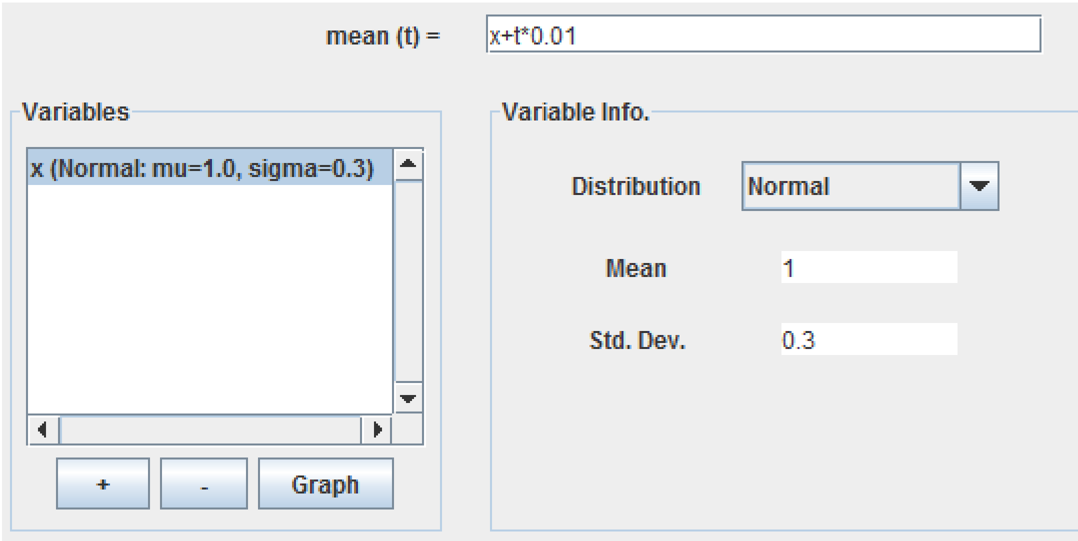
- **Distribution type:** The resistance after maintenance may have three types of distributions: normal, lognormal, or uniform. Such distribution will have a mean of R=100% and a standard deviation of COV*mean. Refer to the Probability Distribution Types section for more information on the Normal, Lognormal, and Uniform distributions.
- **COV:** the Coefficient of Variation is simply the standard deviation of the distribution divided by the mean.
- **R can be greater than 100%:** when this checkbox is activated, resistance after maintenance can exceed the R=100% value. When left unchecked, reconstruction always generates values at or below 100%.

RANDOM MU/SIGMA



The Random Mu/Sigma option provides advanced settings to define the random behavior of system resistance after maintenance. The distribution's mean and standard deviation can be a function of time under this scenario.

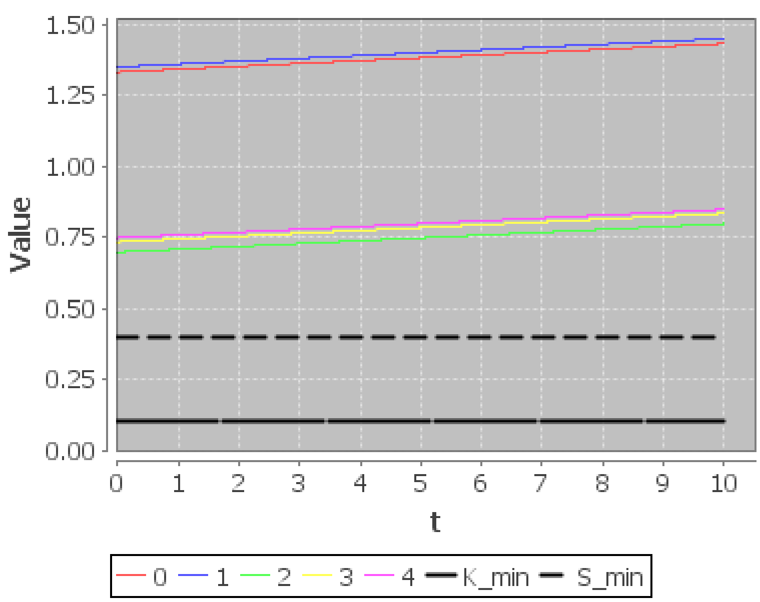
- **Distribution type:** The resistance after maintenance may have three types of distributions: normal, lognormal, or uniform. Refer to the Probability Distribution Types section for more information on the Normal, Lognormal, and Uniform distributions.
- **Mean:** The mean of the distribution can be a function of time. Here's an example:



Note that the variable “t” doesn’t have to be declared in the “variables” list.

Refer to the Equation Editor section for information on supported functions and equation syntax in general.

The equation above is appropriate for a normalized scale, considering that it tends towards R=1.0 when t=0. The graph of this function is shown below:



Each colored line shows the function for a realization of the random variables involved (in this case, x is the only random variable present). The graph shows that the slope of the mean as a function of time is always the same, but the resistances themselves depend on the value of “x”.

- **Std. Dev.:** the standard deviation of the distribution can also be declared to be a function of time, where the variable “t” doesn’t have to be declared in the variables list. Refer to the Equation Editor section for information on supported functions and equation syntax in general.

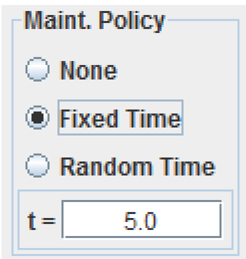
MAINTENANCE POLICY

When running a “successive reconstruction” model, a maintenance policy can be specified.

NONE

If no maintenance policy is in place, the system will be reconstructed if and only if its resistance, R, reaches a threshold value (S_min or K_min depending on the preferences set in the “**Reconstruct”** section).

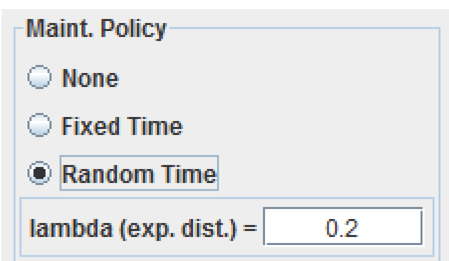
FIXED TIME



Under a “fixed time” policy, the following set of rules is applied:

- If the system fails before the “fixed time” has passed since the last reconstruction (or t=0), the system is reconstructed.
- If the “fixed time” has passed since the last reconstruction (or t=0), the system is reconstructed regardless of its current resistance.

RANDOM TIME

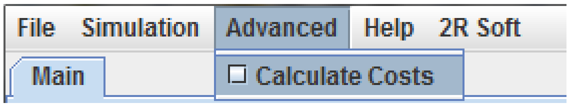


Under a “random time” policy, the following set of rules is applied:

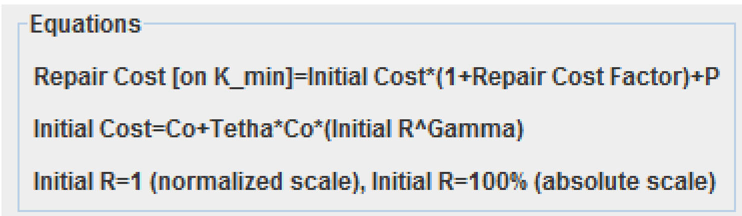
- Maintenance times are randomly generated using an exponential distribution with the specified lambda. Refer to the Probability Distribution Types section for more information on the Exponential distribution.
- If the system fails before the latest generated “random time” has passed since the last reconstruction (or t=0), the system is reconstructed.
- If the latest generated “random time” has passed since the last reconstruction (or t=0), the system is reconstructed regardless of its current resistance.

ADVANCED INPUT – COSTS AND BENEFIT

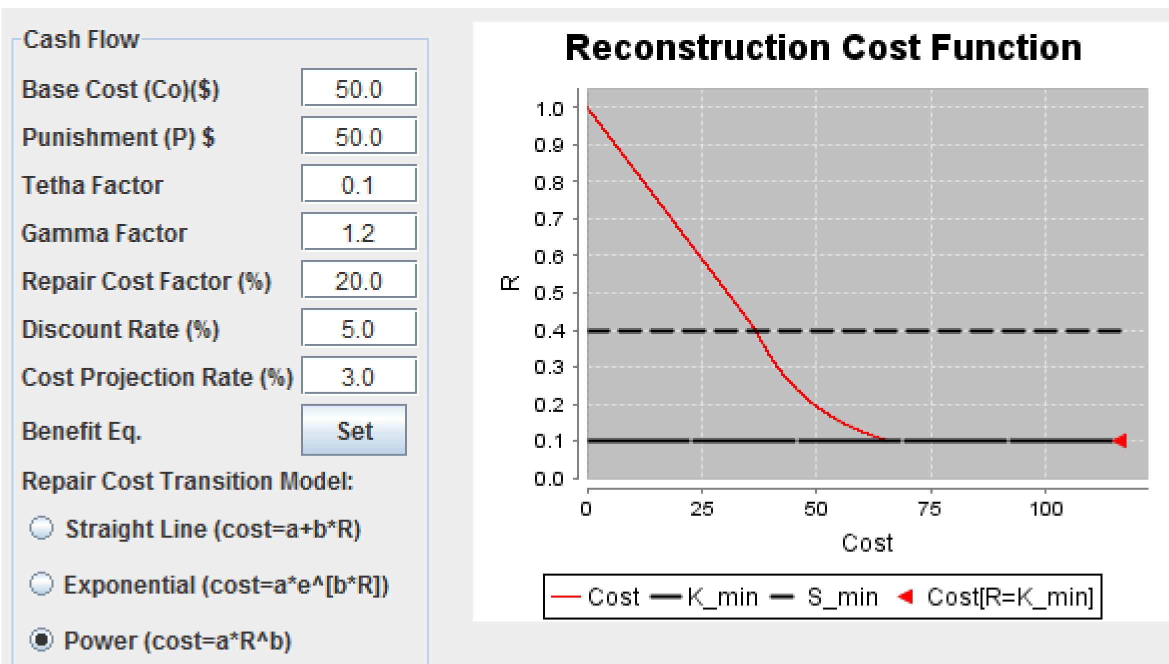
Advanced input is enabled by selecting the "Calculate Costs" option in the "Advanced" menu located at the top of the main window:



The equations that determine the initial and repair costs are as follows:



Various input values are used to generate the cost function:



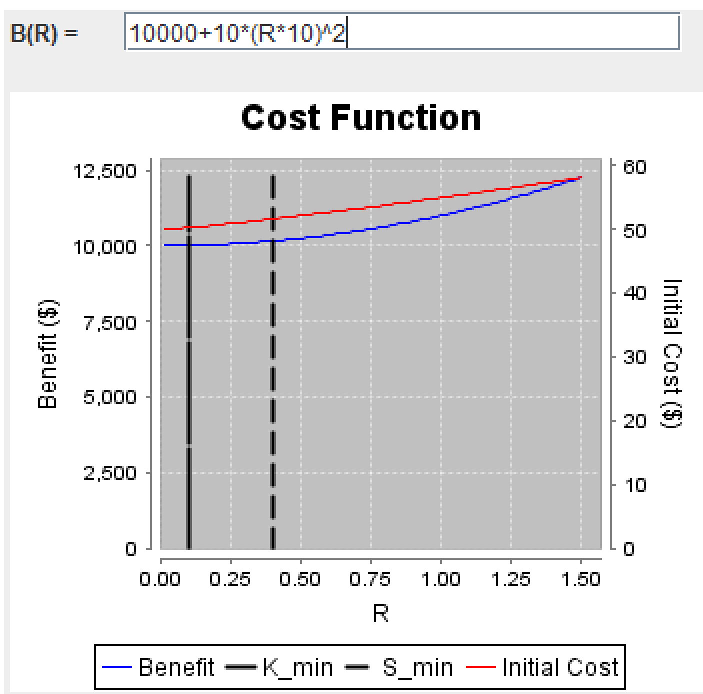
- **Base Cost:** the base cost directly affects the initial cost of the system and, consequently, the repair (or maintenance) cost.
- **Punishment:** the punishment cost <u>is only taken into account when the system is reconstructed at R=K_min (system failure)</u>. It is a penalty to be paid for waiting until the system fails to take corrective action.
- **Tetha Factor and Gamma Factor:** the tetha and gamma factors directly affect the initial cost of the system and, consequently, the repair (or maintenance) cost.

- **Repair Cost Factor:** this factor is a percentage. It indicates the cost overhead associated with the reconstruction of a system that has reached failure (R=K_min).
- **Repair Cost Transition Model:** this setting indicates the type of curve that is to be used to connect the cost of reconstruction at R=S_min with the cost of reconstruction at R=K_min (excluding the punishment). The cost of maintenance at R=S_min is calculated as:

$$Cost\ of\ Maintenance[R = S_{min}] = \frac{Initial\ R - S\_min}{Initial\ R - K\_min} * Initial\ Cost$$

The equation shows that the cost of reconstruction at S_min is calculated as a portion of the initial cost assuming that the value of the system degrades in a linear way with a decrease in its resistance.

- **Discount Rate:** the discount rate is the rate used to calculate the net present value of all of the cash flows occurring at t > 0. This is usually the investor's discount rate or the WACC of the company.
- **Cost Projection Rate:** the cost projection rate is the rate at which prices are expected to increase with time and is used to calculate the cost of the simulated reconstructions when t > 0. This can be viewed as the "expected inflation".
- **Benefit Equation:** the benefit equation is a function of the system's resistance. It stands for the monetary benefit ($) that is attained in the lifetime of a system with initial resistance "R". Refer to the Equation Editor section for information on supported functions and equation syntax in general. An example is shown below:

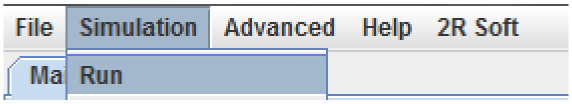


Note that the system resistance, "R", is used as part of the equation.

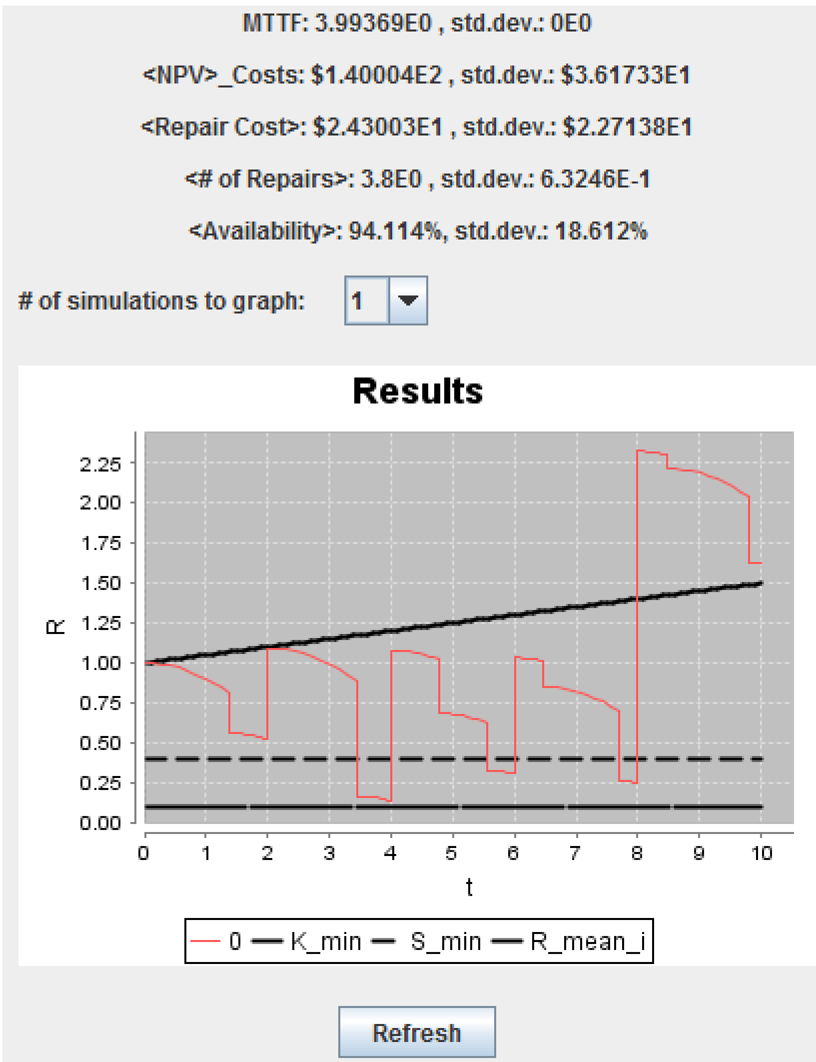
The benefit (blue) is juxtaposed with the initial cost (red) to get a rough idea of their relative proportions. Nonetheless, **the benefit function accounts for an entire lifetime, while the initial cost affects each and every reconstruction throughout the system's lifetime**. For this reason, the benefit is expected to be considerably larger than the initial cost for a positive net benefit to be generated.

RESULTS

A model can be run by selecting the “Run” option from the “Simulation” menu:



The user will be prompted to enter the number of simulations to be carried out. After the simulations are completed, the “Results” tab shows a wide array of information:



- **MTTR or MTTF:** MTTR stands for “Mean Time to Reconstruction” and MTTF stands for “Mean Time to Failure”.
- **MTTF:** MTTF indicates the average time elapsed from reconstruction (or t=0) until system failure and is shown when the “Abandon After First Failure” approach is used or when reconstruction is limited to “R=K_min” in a “Successive Reconstruction” scenario. **Reconstructions resulting from maintenance policy aren’t taken into account.**

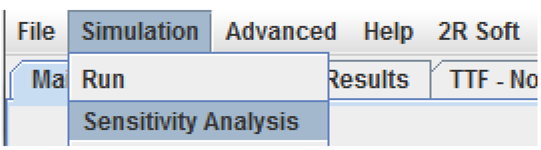
- **MTTR:** MTTR indicates the average time between two consecutive system reconstructions and is shown when the "Successive Reconstruction" approach is used. **Reconstructions resulting from maintenance policy aren't taken into account.**
- **<NPV>_costs:** this statistic is **only shown when "Calculate Costs" is activated in the "Advanced" menu**. It indicates the average net present value of the costs associated with the system, including the acquisition cost and all of the reconstructions.
- **<Repair Cost>:** this statistic is **only shown when "Calculate Costs" is activated in the "Advanced" menu**. It indicates the average cost for a system repair **in current prices (without projecting the cost to t=0)**.
- **<# of Repairs>:** this statistic indicates the average amount of reconstructions that the system receives in its lifetime.
- **<Availability>:** availability is calculated as follows:

$$Availability = \frac{Total\ Time - Down\ Time}{Total\ Time}$$

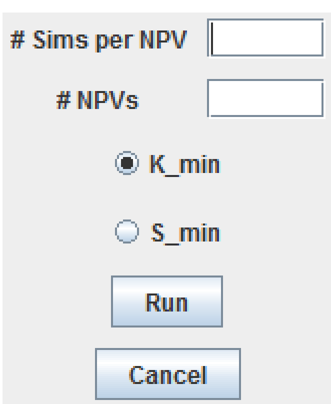
"Down Time" is calculated as the total repair time through a system lifecycle. Thus, availability depends on the **Repair Time (If R=K_min)** setting.

SENSITIVITY ANALYSIS

Sensitivity Analysis is used to investigate the influence that the value of either S_min or K_min has over the average net present value of the costs associated with the system (including acquisition cost and all of the reconstructions). This type of analysis is run by selecting the "Sensitivity Analysis" option from the "Simulation" menu:



OPTIONS



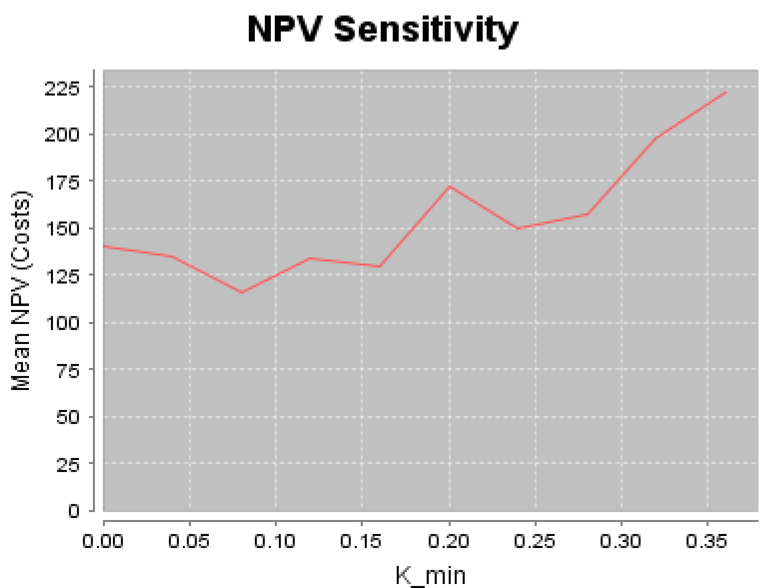
- **# Sims per NPV:** the number of simulations to be run in order to calculate the average NPV of the costs for a single value of K_min or S_min.

- **# NPVs:** the number of values of K_min or S_min to test.
- **K_min:** if K_min is selected, then K_min will be varying between 0 and S_min in regular intervals to calculate the different NPVs.
- **S_min:** if S_min is selected, then S_min will be varying between K_min and R=100% in regular intervals to calculate the different NPVs.

Clearly, the total amount of simulations that will be run is (# Sims per NPV)*(# NPVs).

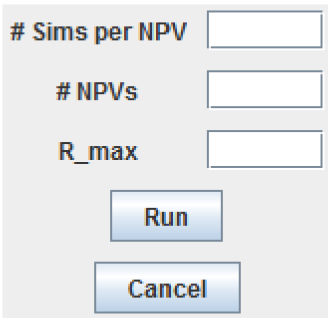
RESULTS

A graph will show how the mean NPV of the costs varies in accordance with the value of K_min or S_min:



COST OPTIMIZATION

The **Cost Optimization** option can only be run on models with an absolute scale, as it analyzes the effect of maximum resistance, R=100%, on the net present value of the costs associated with the system (including acquisition cost and all of the reconstructions).

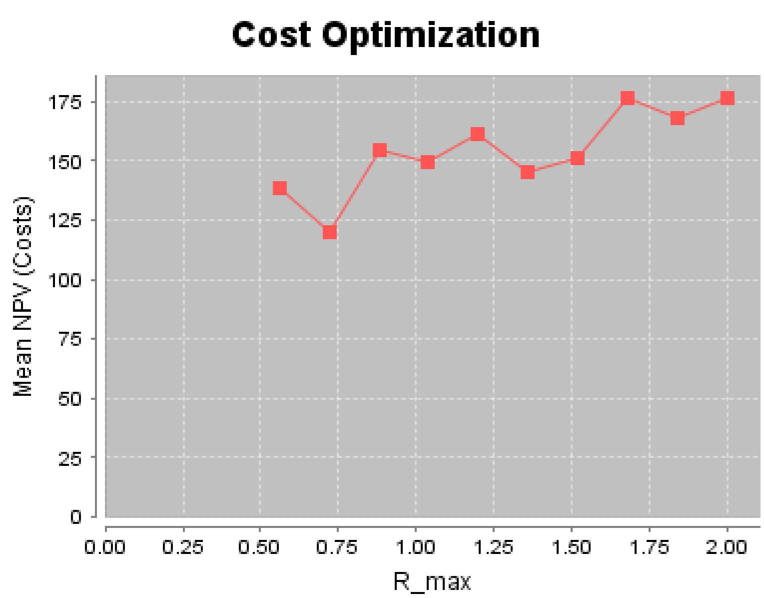


- **# Sims per NPV:** the number of simulations to be run in order to calculate the average NPV of the costs for a single value of maximum resistance, R=100%.
- **# NPVs:** the number of values of maximum resistance, R=100%, to test.
- **R_max:** the highest value of maximum resistance, R=100%, to test. Maximum resistance, R=100%, will be varying between **S_min** and **R_max** in regular intervals to calculate the different NPVs.

Clearly, the total amount of simulations that will be run is (# Sims per NPV)*(# NPVs).

RESULTS

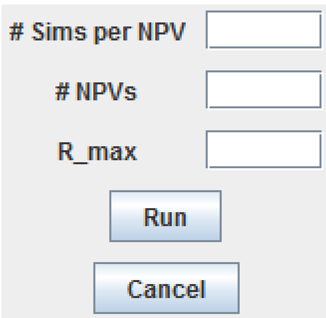
A graph will show how the mean NPV of the costs varies in accordance with the value of maximum resistance, R=100%:



PROFIT OPTIMIZATION

The **Profit Optimization** option can only be run on models with an absolute scale, as it analyzes the effect of maximum resistance, R=100%, on the net benefit associated with the system (including acquisition cost and all of the reconstructions). Net benefit is calculated as follows:

$$Net\ Benefit = Benefit\ [Initial\ R] - NPV\ of\ Costs$$

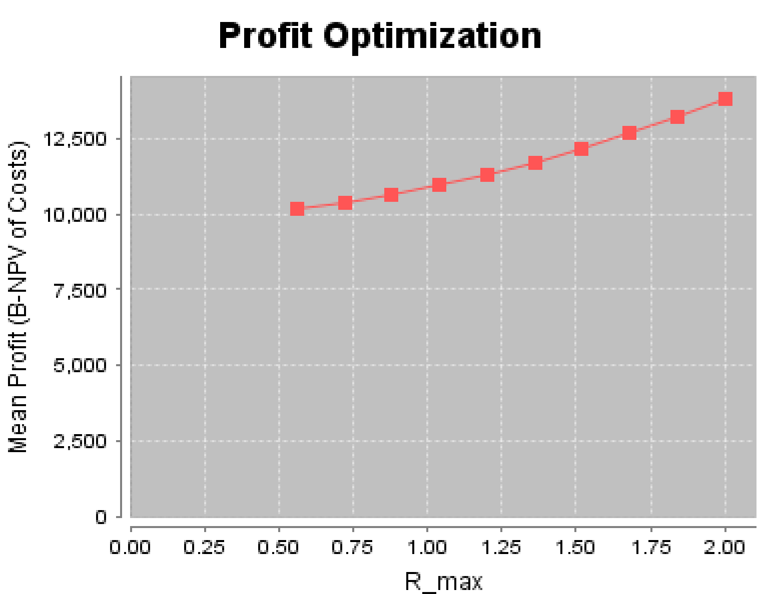


- **# Sims per NPV:** the number of simulations to be run in order to calculate the average net benefit for a single value of maximum resistance, R=100%.
- **# NPVs:** the number of values of maximum resistance, R=100%, to test.
- **R_max:** the highest value of maximum resistance, R=100%, to test. Maximum resistance, R=100%, will be varying between **S_min** and **R_max** in regular intervals to calculate the different net benefits.

Clearly, the total amount of simulations that will be run is (# Sims per NPV)*(# NPVs).

RESULTS

A graph will show how the mean net benefit varies in accordance with the value of maximum resistance, R=100%:



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