Resilience Optimization of Systems of Interdependent Networks

Andrés D. González

School of Engineering

Universidad de los Andes

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1. Reviewer: Prof. Andrew J. Schaefer, Rice University
2. Reviewer: Prof. Ilinca Stanciulescu, Rice University
3. Reviewer: Prof. Kash A. Barker, The University of Oklahoma
4. Co-advisor: Prof. Leonardo Dueñas-Osorio, Rice University
5. Co-advisor: Prof. Andrés L. Medaglia, Universidad de los Andes
6. Advisor: Prof. Mauricio Sánchez-Silva, Universidad de los Andes

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The proposed research presented in this thesis was developed within the Ph.D. programs of the Civil & Environmental Engineering Department at Rice University (Houston, Texas, USA) and the School of Engineering at Universidad de los Andes (Bogotá, Colombia).

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To my beloved parents David and Concepción,

my siblings Julio and Lilián,

and my faithful companion Carolina.

ABSTRACT

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by

Andrés D. González

Critical infrastructure systems such as water, gas, power, telecommunications, and transportation networks, among others, are constantly stressed by aging and natural disasters. Just since 2001, adverse events such as earthquakes, landslides, and floods, have accounted for economic losses exceeding US$1.68 trillion (UNISDR, 2013). Thus, governments and other stakeholders are giving priority to mitigating the effects of natural disasters over such critical infrastructure systems, especially when considering their increasing vulnerability to interconnectedness. Studying the failure and recovery dynamics of networked systems is an important but complex task, especially when considering the emerging multiplex of networks with underlying interdependencies. In particular, designing optimal mitigation and recovery strategies for networked systems while considering their interdependencies is imperative to enhance their resilience, thus reducing the negative effects of damaging events.

The present thesis describes a comprehensive body of work that focuses on modeling, understanding, and optimizing the resilience of systems of interdependent networks. To approach these concepts, we have introduced the Interdependent Network Design Problem (INDP), which optimizes the resource allocation and recovery strategies of

interdependent networks after a destructive event, while considering limited resources and operational constraints. To solve the INDP, we describe the time-dependent INDP (td-INDP), which finds the least-cost recovery strategy for a system of physically and geographically interdependent systems, while accounting for realistic operational constraints associated with the limited availability of resources and the finite capacity of the system's elements, among others. Additional analytical and heuristic solution strategies are also introduced, in order to extend and enhance the td-INDP solving capabilities. Additionally, we propose diverse methodological approaches to incorporate uncertainty in the modeling and optimization processes. Particularly, we present the stochastic INDP (sINDP), which can be seen as an extension of the td-INDP that considers uncertainty in parameters of the model, such as the costs, demands, and resource availability, among others. Finally, we explore different multidisciplinary techniques to allow modeling decentralized systems, as well as to enable compressing the main recovery dynamics of a system of interdependent networks by using a time-invariant linear recovery operator.

The proposed methodologies enable studying and optimizing pre- and post-event decisions, to improve the performance, reliability, and resilience of systems of coupled infrastructure networks, such that they can better withstand normal demands and damaging hazards. To illustrate each of these methodologies, we study a realistic system of interdependent networks, composed of streamlined versions of the water, power, and gas networks in Shelby County, TN. This problem is of interest, since it does not only describe physical and geographical interdependencies, but is also subject to earthquake hazards due to its proximity to the New Madrid Seismic Zone (NMSZ). We show that the proposed methodologies represent useful tools for decision makers and stakeholders, which can support optimal mitigation and recovery planning.

Acknowledgment

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The proposed research presented in this thesis was developed within the Ph.D programs of the Civil & Environmental Engineering at Rice University (Houston, Texas, USA) and the School of Engineering at Universidad de los Andes (Bogotá, Colombia). The ideas and contents included in this thesis are partly based on the publications and presentations developed during my Ph.D. studies, which are listed as follows:

Journal papers

- González, A. D., Dueñas-Osorio, L., Sánchez-Silva, M., and Medaglia, A. L. (2016b). The Interdependent Network Design Problem for Optimal Infrastructure System Restoration. *Computer-Aided Civil and Infrastructure Engineering*,

31(5):334–350

- Smith, A., González, A. D., D'Souza, R. M., and Dueñas-Osorio, L. (2016). Interdependent Network Recovery Games. *Journal of Risk Analysis*, (In review)

- González, A. D., Chapman, A., Dueñas-Osorio, L., Mesbahi, M., and D'Souza, R. M. (2017a). Efficient Infrastructure Restoration Strategies using the Recovery Operator. *Computer-Aided Civil and Infrastructure Engineering*, (In review)

- Abolghasem, S., Gómez-Sarmiento, J., Medaglia, A. L., Sarmiento, O. L., González, A. D., Díaz del Castillo, A., Rozo-Casas, J. F., and Jacoby, E. (2017). A DEA-centric decision support system for evaluating Ciclovía-Recreativa programs in the Americas. *Socio-Economic Planning Sciences*, (In press)

- González, A. D., Medaglia, A. L., Schaefer, A. J., Dueñas-Osorio, L. & Sánchez-Silva, M.(2017). The Stochastic Interdependent Network Design Problem. (Working paper)

- González, A. D., Medaglia, A. L., Sánchez-Silva, M. & Dueñas-Osorio, L. (2017). A hybrid algorithm to solve Interdependent Network Design and Recovery Problems.(Working paper)

- González, A. D., Sánchez-Silva, M., Medaglia, A. L., & Dueñas-Osorio, L. (2017). Optimal resilience-based ranking for Interdependent Networked Systems. (Working paper)

- Chapman, A., González, A.D., Mesbahi, M., Dueñas-Osorio, L. & D'Souza, R., (2017). Understanding Restoration Dynamics and Controllability of Interdependent Networks using System Identification. (Working paper)

- González, A. D., Stanciulescu, I. & Dueñas-Osorio, L. (2017) Fractal based non-linear FEM analysis for trees bending under wind loading. (Working paper)

Peer-reviewed proceedings

- González, A. D., Dueñas-osorio, L., Sánchez-Silva, M., Medaglia, A. L., and Schaefer, A. J. (2017b). Optimizing the Resilience of Infrastructure Systems under Uncertainty using the Interdependent Network Design Problem. In *12th International Conference on Structural Safety & Reliability (ICOSSAR2017)*, pages 1–10, Vienna, Austria

- Chapman, A., González, A. D., Mesbahi, M., Dueñas-Osorio, L., and D'Souza, R. M. (2017). Data-guided Control: Clustering , Graph Products, and Decentralized Control. In *56th IEEE Conference on Decision and Control (CDC2017)*, pages 1–8, Melbourne, Australia

- *González, A. D., Dueñas-Osorio, L., Medaglia, A. L., and Sánchez-Silva, M. (2016a). The time-dependent interdependent network design problem (td-INDP) and the evaluation of multi-system recovery strategies in polynomial time. In Huang, H., Li, J., Zhang, J., and Chen, J., editors, *The 6th Asian-Pacific Symposium on Structural Reliability and its Applications*, pages 544–550, Shanghai, China

- González, A. D., Dueñas-Osorio, L., Sánchez-Silva, M., and Medaglia, A. L. (2015). The Computational Complexity of Probabilistic Interdependent Network Design Problems. In *Proceedings of the 12th International Conference on Applications of Statistics and Probability in Civil Engineering (ICASP12)*, pages 1–8, Vancouver, Canada

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- González, A. D., Sánchez-Silva, M., Dueñas-Osorio, L., and Medaglia, A. L. (2014b). Mitigation Strategies for Lifeline Systems Based on the Interdependent Network Design Problem. In Beer, M., Au, S.-K., and Hail, J. W., editors, *Vulnerability, Uncertainty, and Risk: Quantification, Mitigation, and Management*, pages 762–771. American Society of Civil Engineers (ASCE)

- González, A. D., Dueñas-Osorio, L., Medaglia, A. L., and Sánchez-Silva, M. (2014a). Resource allocation for infrastructure networks within the context of disaster management. In Deodatis, G., Ellingwood, B., and Frangopol, D., editors, *Safety, Reliability, Risk and Life-Cycle Performance of Structures and Infrastructures*, pages 639–646, New York, USA. CRC Press

Technical reports

- [†]Dueñas-Osorio, L., González, A. D., Shepherd, K., and Paredes-Toro, R. (2015). Performance and Restoration Goals across Interdependent Critical Infrastructure Systems. Technical report, National Institute of Standards and Technology (NIST), Houston, TX, USA

Technical talks

- González, A. D., Dueñas-Osorio, L., Medaglia, A.L., Sánchez-Silva, M., & Schaefer, A. (2016). *The Stochastic Interdependent Network Design Problem.* In Annual Meeting of the Institute for Operations Research and the Management Sciences (INFORMS). Nashville, TN, USA.

- González, A. D., Dueñas-Osorio, L., Medaglia, A. L., & Sánchez-Silva, M. (2016). *A hybrid algorithm to solve the time-dependent Interdependent Network Design*

[†]Chapter 8 of National Institute of Standards and Technology (NIST) (2016)

Problem. In Probabilistic Mechanics & Reliability Conference 2016 (PMC2016). Nashville, TN, USA.

- González, A. D., Dueñas-Osorio, L., Medaglia, A. L., & Sánchez-Silva, M. (2015). *Resilience Optimization as an Interdependent Network Design Problem*. In MPE 2013+ Workshop on Natural Disasters, from the Center for Discrete Mathematics & Theoretical Computer Science (DIMACS). Atlanta, GA, USA.

- González, A. D., Dueñas-Osorio, L., Sánchez-Silva, M., & Medaglia, A. L. (2015). *Recovery Dynamics based on the simultaneous use of System Identification and the Interdependent Network Design Problem*. In Workshop on Information Engines / Control of Interdependent Networks. Santa Fe, NM, USA.

- González, A. D., Medaglia, A. L., Dueñas-Osorio, L., & Sánchez-Silva, M. (2015). *Efficient Resilience Optimization of Interdependent Networks*. In Annual Meeting of the Institute for Operations Research and the Management Sciences (INFORMS). Philadelphia, PA, USA.

- González, A. D., Medaglia, A. L., Dueñas-Osorio, L., & Sánchez-Silva, M. (2014). *Improving the Computational Efficiency of the Interdependent Network Design Problem MIP Model*. In Annual Meeting of the Institute for Operations Research and the Management Sciences (INFORMS). San Francisco, CA, USA.

- González, A. D., Dueñas-Osorio, L., Medaglia, A. L., & Sánchez-Silva, M. (2014). *Enhanced component ranking based on the Interdependent Network Design Problem*. In Network Science (NetSci) conference. Berkeley, CA, USA.

- González, A. D., Medaglia, A. L., Dueñas-Osorio, L., Sánchez-Silva, M. & Gómez, Camilo (2013). *The effect of surrogate networks on the Interdependent Network*

Design Problem. In Annual Meeting of the Institute for Operations Research and the Management Sciences (INFORMS). Minneapolis, MN, USA.

A more detailed list of the developed publications and technical talks, which includes their respective abstracts, is presented in Appendix B. I would like to acknowledge the journal Computer-Aided Civil and Infrastructure Engineering, the journal of Risk Analysis, John Wiley & Sons, Blackwell Publishing, Taylor & Francis Group, Elsevier, the American Society of Civil Engineers (ASCE), and all other associated publishing companies and subsidiaries, for granting the authorization to use the submitted preprints in this thesis. The definitive versions of the referenced papers are available at www.onlinelibrary.wiley.com, www.blackwell-synergy.com, www.sciencedirect.com, www.taylorandfrancis.com, open.library.ubc.ca, press.tongji.edu.cn, and ascelibrary.org.

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Part I

Understanding the Proposed Research Problem and its Importance

Chapter 1

Introduction

Proper operation of critical infrastructure networks is vital to our society, supporting the well-being of the population, while enabling adequate governance and safety. In contrast, abnormal operation of such systems generates health and security issues, as well as considerable economic loss. Each year, natural events such as earthquakes, storms, and landslides, kill more than 75 000 people and affect more than 200 million people, usually depriving them from food and housing (Van Wassenhove, 2005). Just between 1996 and 2005, two earthquakes in Turkey and Japan accounted for US$95 billion in losses, three floods in China had an associated economic loss of US$46 billion, and one storm in United States accounted for US$125 billion (Sahin, 2011). Similarly, between 2001 and 2011 adverse natural events have generated economic losses larger than US$1.68 trillion (UNISDR, 2013), where just in 2011 the disaster generated by the Tohoku earthquake and tsunami in Japan caused a direct economic loss of more than US$171 billion, damaging 59 806 buildings and killing more than 13 000 people (Norio et al., 2012). Thus, developing adequate mitigation strategies becomes imperative (Nagasaka, 2008), along with effective relief methodologies and recovery planning for post-disaster response scenarios (Clay Whybark, 2007; Wrobel and Wrobel, 2009).

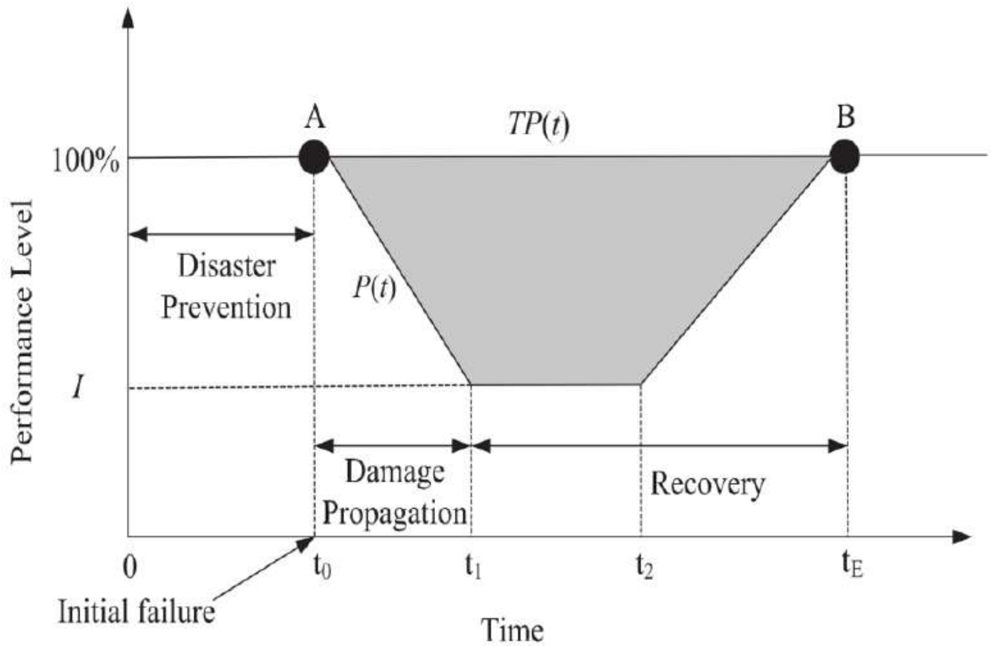


Figure 1.1 : Graphical definition of resilience (Ouyang and Dueñas-Osorio, 2012)

1.1 Defining and optimizing resilience

Reducing the vulnerability of a system while increasing its recoverability can be captured by the concept of maximizing its resilience. There is no universal definition of resilience, but literature predominantly describes it as the ability or capacity of a system to withstand a hazard, contain its damage, and return to adequate performance levels (Ouyang et al., 2012). Figure 1.1 depicts these three aspects of resilience for a given system. This Figure shows a superposition of the performance level associated with a system that is fully operational [$TP(t)$], and the performance level when the system is affected by a given disaster [$P(t)$]. Once the disaster occurs (point A at time t_0), the damage and subsequent negative effects on the performance will propagate trough the system (until a time t_1). After that, the system invests time to gather information regarding the disaster and prepare for the recovery process (from t_1 to

t_2). The recovery process (which takes place from t_2 to t_E), is carried out until the system reaches a desired performance level (point B, at time t_E). Figure 1.1 depicts a 'recovered' level at 100% performance, but in general such a level could be associated with performances above or below 100%. Following this notation, Ouyang and Dueñas-Osorio (2012) define resilience $[R(T)]$ as the ratio between the areas, from times 0 to T, below the performance function associated with a fully operational system $[TP(t)]$, and the performance function associated with a system affected by a disaster $[P(t)]$, i.e.,

$$R(T) \equiv \frac{\int_0^T P(t)dt}{\int_0^T TP(t)dt}. \tag{1.1}$$

Assuming that $TP(t)$ is given and fixed, the resilience $R(T)$ is maximized when $\int_0^T P(t)dt$ is maximized. Moreover, if $TP(t)$ is assumed to be 100% for all times t, then $\int_0^T TP(t)dt = T$, and

$$R(T) = \frac{\int_0^T P(t)dt}{T} = \bar{P}(T), \tag{1.2}$$

where $\bar{P}(T)$ represents the average of $P(t)$ from times 0 to T.

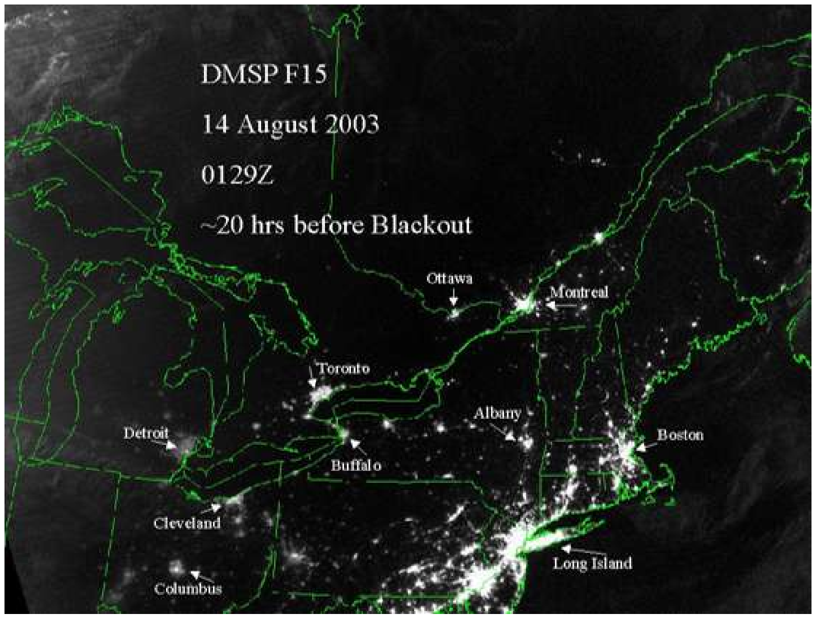
Unfortunately, studying the resilience of realistic networked systems presents multiple challenges associated with, among others, their constant expansion, both in space and demand, and their increased interdependency, which directly impacts their vulnerability and recoverability (Dueñas-Osorio, 2005; Hernandez-Fajardo and Dueñas-Osorio, 2011). Moreover, for realistic applications, it is important to be able to not only quantify the resilience of the studied systems, but also to determine the decisions that maximize it (Ouyang and Dueñas-Osorio, 2012; Hernandez-Fajardo and Dueñas-Osorio, 2013). By maximizing the resilience of real systems, these will be able to both better endure diverse hazards and recover faster, ideally with minimum economic and social impact.

1.2 Systems of interdependent networks

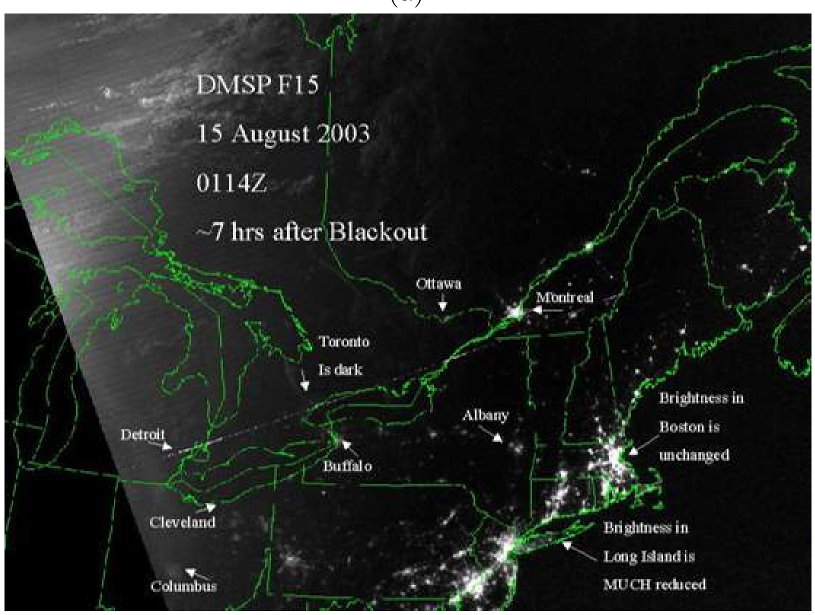
One of the most prevalent characteristics of realistic networked systems such as critical infrastructure, is that these systems are often interconnected, and consequently highly interdependent. Thus, modeling infrastructure networks as isolated systems would not reflect important dynamics that impact their operation and performance (Anderson et al., 2007). There are a plethora of examples that illustrate the effects of interconnectedness and interdependencies between infrastructure networks, such as the blackouts in Italy on September of 2003 (Schneider et al., 2013) and North America on August 2013 (Veloza and Santamaria, 2016; Zhang and Peeta, 2011), or the 2011 Japan earthquake and tsunami disaster (Norio et al., 2012), among others. In each case, interdependencies between different infrastructure networks, such as power, water, transportation, and telecommunications, exacerbated the damage propagation and caused cascading failure effects. To illustrate the impact of these events, Figure 1.2 shows satellite images of Northeastern United States and Southeastern Canada, before and after the August 2003 blackout, where it can be seen that, for more than 7 hours after the event, the Long Island region had a major reduction on its power supply, and cities such as Toronto, Ottawa, and Buffalo were almost completely left without power. However, Figure 1.2 only illustrates the effects associated with the power network. The loss of electricity supply also caused, among others, the inoperability of water pumps, which left many people without ready access to potable water (Beatty et al., 2006), and the reduced access and operation of the transportation network, due to lack of signaling and illumination, as well as the nonviable operation of the electricity-based trains (Deblasio et al., 2004). Just in New York City, around 11 600 road intersections collapsed due to complete inoperability of traffic lights and other signaling, and all 413 train sets came to a halt due to lack of power, affecting more that 400 000 train users (Deblasio et al., 2004).

Considering the importance of modeling and understanding interdependencies and their effects on critical infrastructure networks, research on interconnected and interdependent infrastructure systems has gained much interest in recent years. Figure 1.3 illustrates the number of papers written (and their citations), per year, related to interconnected critical infrastructure, showing a growing research interest on interdependent networks, despite being a relatively novel subject in the literature.

Early works on interconnected critical infrastructure started by acknowledging and categorizing the diverse interdependecies observed in infrastructure systems (Rinaldi et al., 2001). Later, some works focused on detecting and quantifying the vulnerability caused by such interdependencies (Buldyrev et al., 2010; Vespignani, 2010; Gao et al., 2011; Hernandez-Fajardo and Dueñas-Osorio, 2013), and subsequently on reducing such vulnerability (Brummitt et al., 2012). Simultaneously, other works started focusing on describing and developing recovery strategies for damaged interdependent systems (Lee II et al., 2007; Cavdaroglu et al., 2011). More recently, researchers have focused on integrating these concepts, by studying the resilience of these interdependent networked systems (Pant et al., 2013; Ouyang, 2014; Shafieezadeh et al., 2014; Baroud et al., 2015). However, in order to optimize the resilience of systems of interdependent networks, there are still multiple challenges that need to be addressed.



(a)



(b)

Figure 1.2 : Satellite images taken before (a) and after (b) the August 2003 blackout that affected the Northeastern United States and Southeastern Canada (U.S. National Oceanic and Atmospheric Administration (NOAA), 2003)

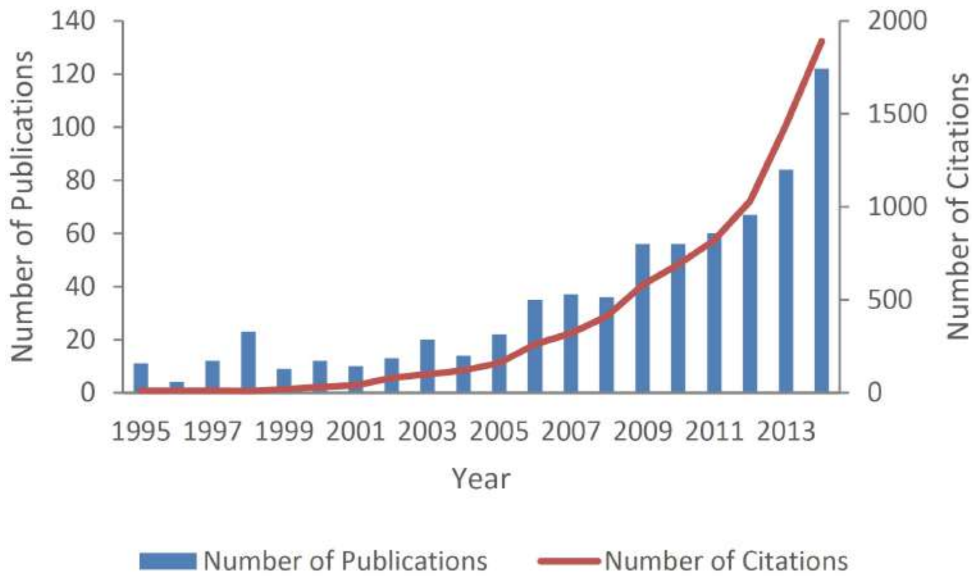


Figure 1.3 : Papers on interconnected critical infrastructure (National Institute of Standards and Technology (NIST), 2016)

1.3 Current challenges

From the diverse literature related to modeling and optimizing the vulnerability, recoverability, and resilience of interdependent networks, the three most prevalent and relevant challenges we observed are:

- Modeling realistic networks and their interdependencies: It is important to adequately define the way in which the networks and their interdependencies are modeled. It is crucial to evaluate to what level of detail we should model the networks, in order to take into account their most important properties and behaviors, while maintaining simplicity and manageability. For real infrastructure systems, it is imperative to include aspects such as the existence of commodities flowing through the networks, being supplied and demanded. Similarly, there must be a limit to the system capacities for its operation. The system of networks should also be subjected to failures while recovering from those failures must be limited by a finite amount of resources.

- Handling large-scale problems: Critical infrastructure systems are increasing both in size and in demand of commodities. Thus, it is imperative that proposed computational models are scalable and susceptible to enhancements, either by analytical or heuristic methods.

- Considering uncertainty: Given that the studied systems are subject to failure and forecasting errors, it is important that the proposed computational models can consider uncertainty. To this end, we must focus on developing fast algorithms that facilitate the use of sampling techniques. Additionally, the proposed algorithms should also be able to embed uncertainty in its paramaters, to guarantee that resultant analyses agree with the expected behavior of the system.

1.4 Research objectives

Considering the preceeding discussion, the main objective of this thesis is *to develop efficient theoretical and computational tools to model, quantify, and optimize the resilience of systems of interdependent networks.*

In particular, the proposed work focuses on developing tools applied to resource allocation and emergency response for critical interdependent infrastructure systems, in a disaster management context, while considering the three challenges previously detailed.

This thesis presents a body of work that focuses on integrally addressing the aforementioned challenges, in order to model and quantify the effects of interdependencies in realistic infrastructure networks, optimize their recovery, and ultimately optimize their overall resilience. By virtue of doing so, the developed models provide useful tools for decision makers and stakeholders, to support pre- and post-event decisions associated with mitigation, resource allocation, and emergency response, among others.

1.5 Thesis structure

Chapter 2 presents a detailed study on how to model and recover networked infrastructures, as well as on how to model their associated interdependencies. Considering the diverse existing approaches, their assumptions, and limitations, we detail what are the desired characteristics and goals that an adequate interpendent-networks recovery problem should consider. In particular, Section 2.2 presents the denominated Interdependent Network Design Problem (INDP), as the problem associated with determining the optimal recovery strategy of a system of interdependent networks, while considering realistic constraints associated with its operation, and its resource availability. Section 2.2 also proposes a mathematical formulation to solve the INDP, denominated the time-dependent INDP model (td-INDP). The td-INDP is a Mixed Integer Program (MIP) formulation, that focuses on minimizing the costs associated with the operation and recovery of a system of interdependent networks, while considering constraints associated with resource availability, flows balance, systems' capacities, and physical and geographical interdependencies, among others.

Chapter 3 explores diverse techniques to improve the computational capabilities of the td-INDP, such that it can be solved faster and can handle larger systems. On one hand, Section 3.1 presents a heuristic approach denominated the iterative INDP (iINDP), which decomposes the studied planning horizon into smaller more manageable time horizons, and solves iteratively for each of them. On the other hand, Section 3.2 presents a comprehensive sensitivity analysis on the td-INDP parameters and its structure. In particular, Section 3.2.2 shows that the td-INDP has a special structure, which makes it suitable for analytical decomposition techniques.

Chapter 4 describes diverse methodological approaches to allow considering diverse sources of uncertainty. Section 4.1 describes the uncertainty associated with the failure modes in the system, and presents a simulation-based study that offers insightful element-wise information to facilitate pre-event decision making support. In particular, we propose diverse resilience metrics that describe different aspects of the studied system, such as the networks' topologies, their flow of commodities, or the specific hazard studied. Then, Section 4.2 details the importance of accounting for the uncertainty related to the system properties, such as its demands and resource availability. To account for this source of uncertainty, we propose a stochastic formulation, denominated the stochastic INDP (sINDP) model, which determines the recovery strategy that minimizes the expected recovery costs.

Chapter 5 presents additional multidisciplinary techniques to further expand the td-INDP capabilities. Section 5.1 describes the importance of considering additional realistic socio-technical aspects when studying multiple interdependent networks, such as the existence of multiple interacting decision makers in charge of different networks in the system. In addition, Section 5.2 presents a methodology that approximates the td-INDP using a linear time-invariant operator, denominated the recovery operator. This operator identifies key properties related to the recovery dynamics, and allows constructing efficient recovery strategies in polynomial time.

Finally, Chapter 6 presents a summary of the main findings from this thesis, and highlights opportunities for future research that build upon the proposed INDP models.

Part II

Addressing Current Modeling Challenges

Chapter 2

The Interdependent Network Design Problem

Properly modeling interdependent networks and their operation is vital to study and understand the key factors associated with the performance of such systems and their dependence on damaged components*. In order to determine the key properties that need to be modeled, as well as the best approach to model them, the next section focuses on reviewing some of the most relevant literature in the field of infrastructure recovery.

2.1 Recovery of networked infrastructure systems

Considering the importance of ensuring adequate performance of infrastructure systems, this topic has been extensively studied in different fields, such as civil engineering, economics and political sciences, among others. Moreover, there are different methodological approaches used to study these systems, such as dynamical systems (Hallegatte and Ghil, 2008), complex systems (Comfort et al., 2004; Chu, 2009; Nejat and Damnjanovic, 2012), and network analysis (Wang et al., 2013; National Institute of Standards and Technology (NIST), 2016; Gómez et al., 2013; Dueñas-Osorio and Kwasinski, 2012). Considering that the main focus of the proposed research is to optimize the resilience of interdependent networked systems, this literature review is oriented towards the network analysis perspective.

*This Chapter is based on the ideas and contents presented in González et al. (2016b) and González et al. (2016a). The definitive versions of these papers are available at www.onlinelibrary.wiley.com and press.tongji.edu.cn

Viswanath and Peeta (2003) developed a relief distribution algorithm, based on a multicommodity maximal covering network design problem, to maximize the total population covered while minimizing people's travel time. Similarly, Sheu (2007) designed an approach to emergency logistics for quick response to large-scale disasters, based on a hybrid fuzzy clustering optimization problem. The model involves recursive mechanisms associated with disaster-affected areas grouping and relief co-distribution.

Later, Sheu (2010) presented a dynamic model that incorporates imperfect information, considering uncertain and dynamic features of the relief demands. Particularly, the model focuses on treating related inconsistencies from multiple data sources, by applying a data fusion technique. Yi and Kumar (2007) also developed a model for disaster relief operations, using a hybrid optimization approach. The proposed network-flows model solves the multicommodity dispatch to distribution centers, while a metaheuristic based on ant-colony optimization designs the vehicle routing. Stepanov and Smith (2009) developed a multi-objective evacuation routing model based on optimization modeling and queueing theory. Saadatseresht et al. (2009) used a multi-objective evolutionary optimization approach to develop an evacuation strategy; they emphasized on spatial aspects such as safe areas selection.

In order to determine the location and allocation of several kinds of emergency supplies, Rawls and Turnquist (2010) proposed a methodology for pre-positioning emergency supplies based on a heuristic approach. To address large-scale instances, they proposed a structure based on a minimum cost flow problem and solved its Lagrangian relaxation. Bozorgi-Amiri et al. (2011) focused on the logistics associated with the supply of relief commodities and different sources of uncertainty affecting the relief chains; they developed a multi-objective stochastic optimization model. Özdamar and Demir (2012) proposed a hierarchical clustering method that reduces large-scale routing networks into smaller ones, so as to enable linear optimization.

Ganapati (2012) showed the importance of long-term recovery and not only the rapid reconstruction at early stages of the process. Even though he mentions the importance of including the effects of the initial reconstruction phase, since it could lead to the detriment of the long-term recovery goals and priorities, his work focused only on a conceptual approach. Gómez et al. (2013) developed an algorithm based on a hierarchical infrastructure network representation that reduces a realistic complex networked system into a simpler one, while preserving the most significant information and dynamics for effective decision support. However, their approach focused on studying the reliability and propagation of damage on an infrastructure system, without explicit methods to perform optimal recovery processes.

In general, the aforementioned references emphasize the importance of considering the limited access to resources during the emergency-response phase, as well as the limited capacities of each system. However, these works do not consider the effects associated with interdependencies between different infrastructure systems.

Network Interdependencies

Understanding and managing interdependencies among infrastructure systems is imperative to ensure their service continuity after disruptive events (Wallace et al., 2001). In particular, the interdependency of infrastructure networks increases their vulnerability to natural disasters, particularly when such disasters are neither too small nor too large (Dueñas-Osorio et al., 2007b,a). Hence, given that infrastructure systems are increasingly interdependent, interdependence analysis is critical when studying reliability and performance of systems of infrastructure, as well as when designing recovery and mitigation plans (Dueñas-Osorio and Kwasinski, 2012). However, before focusing on developing strategies to study the impact of interdependencies on the performance of a given system, it is important to determine which interdependencies are the most relevant, and what is the specific goal of the developed models to capture them.

On one hand, regarding the types of interdependence between networks that are relevant in infrastructure systems, there are different classifications proposed in the literature. Rinaldi et al. (2001) classify them as physical, cyber, geographic, and logical interdependencies. Lee II et al. (2007) classify them as input, mutual, shared, exclusive or, and co-located interdependencies. For the remaining of the thesis, we will use the classification proposed by Rinaldi et al. (2001), given that it presents a more general description of the interdependencies, which can be adapted to a broader range of situations. Under such classification, physical interdependence refers to the case when the performance of a given network depends on the outputs of other networks. For instance, consider the existing interdependence between the power networks and communication networks, where a power outage may affect some data centers and routers in the latter depending on the extent of backup systems. Cyber interdependence occurs when the interdependence between two networks is based on shared information. The Smart Grid concept (Farhangi, 2010), which refers to how information about supply and demand is automatically used to determine the production and distribution of electricity, is an example of cyber interdependence between the power and the telecommunication networks. Geographic interdependence denotes the cases when a local event can create state changes in several networks simultaneously. For example, fixing an underground water pipeline may affect the transportation network around the intervened area, probably slowing traffic and increasing congestion. Finally, there is logical interdependence when two or more systems are interconnected via control, regulatory, or other mechanism that are not considered physical, geographical or cyber (Setola and Porcellinis, 2009). Logical interdependence can be observed when human decisions affect the state of the networks, for instance when designing a recovery or expansion plan that prioritizes certain networks.

On the other hand, the existing methodologies for interdependent network analysis can be grouped according to their goals as follows: analytical models, performance evaluation models, design methodologies, mitigation models, and recovery methodologies. Analytical and performance evaluation models focus on tools to understand how interdependencies influence the systems behavior, performance, and reliability. For instance, Amin (2002) focused on describing the complexity of networks (such as transportation, power, telecommunications, and financial, among others) and their ever increasing interdependence, vulnerability, and diminished security. Similarly, Zevenbergen et al. (2012) emphasized the importance of understanding and tackling the complex linkages between subsystems and services, and the cascading effects of one subsystem upon another, since this deep relationship is imperative to establish the full benefit and costs of any proposed disaster management strategy.

Design models are helpful to conceptualize and build infrastructure systems, taking under consideration constraints such as the demand of the commodities, limited resources, flow capacities, and costs of construction and transportation. Buldyrev et al. (2010) explored how interdependent networks may collapse under the effect of a catastrophic cascade of failures, due to their increased vulnerability. More specific to infrastructure systems, models such as the ones proposed by Hernandez-Fajardo and Dueñas-Osorio (2011) and Wu and Dueñas-Osorio (2013) quantify how interdependencies impact the failure and performance of infrastructure systems, where the complex connections between different networks lead to emergent response effects on the performance of individual networks.

Mitigation models focus on preparing systems to handle future hazards, so they can reduce the associated risks and enhance their resilience. For example, Yagan and Zhang (2012) described a methodology to optimally allocate links in cyber-physical systems, in order to improve their performance and robustness.

Finally, recovery models focus on taking systems with failing components to a functional state, considering important aspects such as minimizing the recovery time and the associated costs. Focusing on recovery, Lee II et al. (2007) presented a general network flow model for infrastructure restoration using Mixed Integer Linear Programming (MILP). Such a model focuses on determining which components should be reconstructed to recover a damaged system, but does not indicate when to perform the reconstruction jobs. Cavdaroglu and Nurre (2010), Cavdaroglu et al. (2011) and Nurre et al. (2012) used MILP models to study network restoration and scheduling for interdependent infrastructure systems, where an explicit time index is used to assign reconstruction periods, while Sharkey et al. (2015) showed how to account for information sharing when optimizing the recovery process of such interdependent systems. Even though the presented models consider functional interdependencies, they do not consider relevant issues associated with co-location and the availability of different required resources, among others. In this chapter, we propose a recovery model that can be easily expanded to be used for any of the other described goals, while being able to consider co-location and resource-availability constraints.

2.2 The Interdependent Network Design Problem (INDP)

At the core of the proposed research, it is important to define a problem that focuses on optimizing the recovery process of a partially destroyed system, such that the resilience of the system is maximized. This problem, that designs recovery strategies for interdependent systems, will be consequently defined as the Interdependent Network Design Problem (INDP). Nevertheless, there are multiple approaches and considerations that could be made when defining the INDP, depending on how the performance of the system is measured, the properties and characteristics that will be modeled from the real system, and particularly, the constraints that will take place.

2.2.1 What should the INDP consider?

The first aspect that must be defined refers to the objective of the INDP. By definition, the INDP must focus on optimizing the recovery process of a destroyed system, such that its resilience is maximized. Nevertheless, the quantified resilience will depend directly on the performance metric used. In particular, critical infrastructure systems should guarantee adequate operation to the community, that in the case of utilities such as water, gas, and power, relates to the demand that the system is able to satisfy. Thus, it is important that the INDP focuses on maximizing the percentage of demand being supplied. But, even though managers in charge of such systems should have as a priority properly covering the community demands on the diverse offered commodities, they also are concerned about reducing their operation (and reconstruction) costs. Thus, the INDP should focus on maximizing the supply of commodities (as first priority), while also reducing the associated costs of recovery and operation.

Also, infrastructure systems are subject to constraints related to their operation and recovery capacity. Regarding the operation, infrastructure systems will have to supply certain demand of commodities at different locations, and will be subject to limited transportation capabilities. Regarding the recovery process, infrastructure systems must be efficient about their use of the limited resources at hand. Related to both operation and recovery, infrastructure networks usually depend on the proper operation of other networks.

Considering all these aspects, let us define the Interdependent Network Design Problem (INDP) as the problem of finding the least-cost recovery strategy of a partially destroyed system of infrastructure networks, subject to budget, resources, and operational constraints, while considering interdependencies between the networks, such as physical and geographical (Rinaldi et al., 2001). Under such a definition, the reconstruction costs can include the costs of the resources used in the reconstruction, labor costs, the costs of preparing the geographical locations for the reconstruction process, and costs associated with the system not being able to perform adequately (González et al., 2016b).

Based on the previous discussion, in order to solve the INDP for generalized practical applications, we propose formulating a Mixed Integer Programming (MIP) model with the following features:

- Models a system of interdependent networks with multiple commodities, demands, capacities, and functional and reconstruction constraints.
- Considers multiple network interdependencies (e.g., physical, geographical, etc.).
- Analyzes the impact of the interdependencies on the networks performance and operative costs.
- Indicates the optimal time-dependent recovery strategy for a system of interdependent networks after a damaging event.

The proposed MIP model, denominated the time-dependent INDP (td-INDP), is based on the following assumptions, associated with the flows and capacities of the system of networks, the costs related to the operation and restoration of the system, and the interdependencies studied:

Network flows

- Each infrastructure network is composed by arcs and nodes subject to failure, which can be repaired or reconstructed.

- Each infrastructure network transports one or several commodities, where there is a known demand or supply pattern for each commodity associated with each infrastructure network.

- Each commodity flows through only one infrastructure network.

- There is a known flow capacity for each arc in every infrastructure network, shared by commodities flowing through that network.

- The flow capacities are independent between different infrastructure networks since they transport different commodities.

Costs

- There is a known fixed cost associated with the lack of supply of each commodity.

- The flow costs for each commodity and the reconstruction costs for each arc in each network are known and fixed.

- The reconstruction process involves costs for repairing the components (nodes and arcs) and for preparing the reconstruction locations.

- All costs should be in equivalent units, and should reflect their real values to the cost bearers, such that they already consider possible weighting skews among them.

Interdependencies

- Infrastructure networks may be physically interdependent, i.e., the failure of some nodes imply that other nodes may not function.

- Several infrastructure networks may have geographical interdependence, i.e., components (nodes and arcs) sharing physical space. When there are different network components that have to be fixed, the cost of preparing the overlapping locations is shared between the networks.

Based on these assumptions, the proposed td-INDP model uses as an input the information associated with a partially destroyed system of interconnected infrastructure networks (costs, capacities, damaged components, etc), and returns the recovery process that maximizes the resilience of the system. In particular, the proposed td-INDP formulation is structured as follows.

2.2.2 td-INDP mathematical formulation

Let $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ be a directed network defined by a set $\mathcal{N}$ of nodes, and a set $\mathcal{A}$ of arcs, which represents the complete system of individual infrastructure networks. Let $\mathcal{K}$ be the set of infrastructure networks, such as electrical, water, and telecommunication, among others. Likewise, let $\mathcal{L}$ be the set of commodities flowing through the networks. For each infrastructure network $k \in \mathcal{K}$ there is a related subset $\mathcal{L}_k \subseteq \mathcal{L}$, indicating which commodities flow through network k. From the assumptions, note that $\cup_{k\in K}\mathcal{L}_k = \mathcal{L}$ and that $\mathcal{L}_k \cap \mathcal{L}_{\tilde{k}} = \emptyset, \forall k, \tilde{k} \in \mathcal{K} \mid k \neq \tilde{k}$. Also, for each infrastructure network $k \in \mathcal{K}$ there is an associated subgraph $\mathcal{G}_k = (\mathcal{N}_k, \mathcal{A}_k)$, where $\mathcal{N}_k$ and $\mathcal{A}_k$ are respectively the sets of nodes and arcs associated with that specific network. It should be noted that $\cup_{k\in K}\mathcal{N}_k = \mathcal{N}$ and $\cup_{k\in K}\mathcal{A}_k = A$. Let $\mathcal{N}'_k \subseteq \mathcal{N}_k$ and $\mathcal{A}'_k \subseteq \mathcal{A}_k$ denote the set of destroyed nodes and arcs in each network $k \in \mathcal{K}$ after a disaster. Correspondingly, $N' = \cup_{k\in K}\mathcal{N}'_k$ and $A' = \cup_{k\in K}\mathcal{A}'_k$ denote the set of nodes and arcs destroyed in the entire system of systems. Also, for each network $k \in \mathcal{K}$ the set $\mathcal{N}^*_k \subseteq \mathcal{N}_k$ indicates the nodes that require full satisfaction of its demand to become functional.

Now, assume that there is limited time horizon to perform the full recovery of the system, divided into T time periods. Define $\mathcal{T} = \{0, 1, .., T\}$ as the set of periods in which the problem is defined, which means that the parameters and variables will depend on the time $t \in \mathcal{T}$. Time $t = 0$ indicates the moment right after the occurrence of the damaging event, and times $t \in T \setminus \{0\}$ indicate the periods in which the recovery process takes place. For every period $t \in \mathcal{T}$, infrastructure network $k \in \mathcal{K}$ and node $i \in \mathcal{N}_k$, let b_{iklt} be the demand or supply for commodity $l \in \mathcal{L}_k$ (if $b_{iklt} > 0$, i is a supply node; if $b_{iklt} < 0$, i is a demand node; and if $b_{iklt} = 0$, i is a transshipment node). The associated reconstruction cost of node i in network k is denoted by q_{ikt}. For every arc $(i, j) \in \mathcal{A}_k$ in network $k \in \mathcal{K}$, we define a reconstruction cost f_{ijkt}, an associated capacity denoted by u_{ijkt}, and a cost c_{ijklt} that represents the cost per unit flow of commodity $l \in \mathcal{L}_k$. The parameter M^-_{iklt} represents the unit cost of commodity $l \in \mathcal{L}_k$ that fails to be supplied in node $i \in \mathcal{N}_k$ of network $k \in K$ in period $t \in \mathcal{T}$; similarly, M^+_{iklt} represents the corresponding cost of excess supply.

The td-INDP model also considers a set of limited resources $\mathcal{R}$ involved in the restoration process (e.g., budget, time, workforce) that prevents immediate total reconstruction; hence, the availability of resource $r \in \mathcal{R}$ is represented by v_r. For every period $t \in \mathcal{T}$, node $i \in \mathcal{N}_k$ and arc $(i, j) \in \mathcal{A}_k$ in network $k \in \mathcal{K}$, let p_{ikrt} and h_{ijkrt} be the amount of resource $r \in \mathcal{R}$ that the recovery of every node and arc would require, respectively. In a general sense, the physical interdependence used in the model is based on the idea that the functionality of a node $i \in \mathcal{N}_k$ is related to the functionality of another node $j \in \mathcal{N}_{\tilde{k}}$, where $k, \tilde{k} \in \mathcal{K}$. To model the interdependence between any pair of nodes, for every period $t \in \mathcal{T}$ there is a parameter $\gamma_{ijk\tilde{k}t}$ that relates them. The parameter $\gamma_{ijk\tilde{k}t}$ allows the td-INDP model to account for four different cases of physical interdependencies. The first case of physical interdependence is when node $j \in \mathcal{N}_{\tilde{k}}$ can be functional only if another specific node $i^* \in \mathcal{N}_k$ is functional; then $\gamma_{ijk\tilde{k}t}$ is 1 when $i = i^*$, and 0 otherwise. The second case of interdependence is when node

$j \in \mathcal{N}_{\tilde{k}}$ can be functional only if there is at least one functional node from a subset $\mathcal{N}_k^* \subseteq \mathcal{N}_k$; then $\gamma_{ijk\tilde{k}t}$ is 1 for $i \in \mathcal{N}_k^*$, and 0 otherwise. The third case is when node $j \in \mathcal{N}_{\tilde{k}}$ can be functional only if every node from a subset $\mathcal{N}_k^* \subseteq \mathcal{N}_k$ is functional; then $\gamma_{ijk\tilde{k}t}$ takes the value of $1/|\mathcal{N}_k^*|$ for $i \in \mathcal{N}_k^*$, and 0 otherwise. The fourth, and most general case, is when node $j \in \mathcal{N}_{\tilde{k}}$ depends partially on the functionality of the nodes from a subset $\mathcal{N}_k^* \subseteq \mathcal{N}_k$, where the dependence on each of them is not the same, i.e., each node from $\mathcal{N}_k^*$ provides only a fraction of functionality to node j; then $\gamma_{ijk\tilde{k}t}$ is the fraction of functionality that each node $i \in \mathcal{N}_k^*$ provides to node j.

The interdependencies based on co-location assume that repairing one specific arc in a network implies preparing a shared space with other networks. Preparing a space refers to construction tasks required to access and repair elements of the system. These tasks may include debris removal, excavation, shoring, soil improvement, temporary detours, etc. Let $\mathcal{S}$ be the set of geographical spaces, where at each period $t \in \mathcal{T}$, each $s \in \mathcal{S}$ has an associated cost of preparation g_{st}. There is a binary parameter α_{ikst} that takes the value of 1 if node $i \in \mathcal{N}_k$ of network k falls inside space s; it takes the value of 0, otherwise. There is also a binary parameter β_{ijkst}, which takes a value of 1 if for network k, arc $(i, j) \in \mathcal{A}_k$ (or a part of it) is inside space s; it takes the value of 0, otherwise. Hence, this parameter indicates whether the space s has to be prepared or not when fixing the arc $(i, j) \in \mathcal{A}_k$. If a given space s has to be prepared due to the reconstruction of one or several arcs, or due to one or several nodes, then g_{st} is defined as the maximum of those individual preparation costs, and is due only once. Note that the set of spaces $\mathcal{S}$ refers to a partition of the geographical extent that contains the complete interdependent infrastructure system, but the size and shape of each space may vary according to what each space is desired to represent. For example, if we analyze an infrastructure network in a given county, $\mathcal{S}$ may represent the political districts (e.g., by towns or neighborhoods), the service areas of each system component, a geographical distribution based on the hazard levels due to floods or earthquakes, and so on. Nevertheless the set $\mathcal{S}$ must always be a mutually exclusive and collectively exhaustive collection that contains the system of infrastructure networks.

The decision variable denoted by x_{ijklt} represents the flow of commodity $l \in \mathcal{L}_k$ through arc $(i,j) \in \mathcal{A}_k$ in network $k \in \mathcal{K}$ at period $t \in \mathcal{T}$. For each arc $(i,j) \in \mathcal{A}'_k$ in network $k \in \mathcal{K}$ there is also a binary variable Δy_{ijkt} that takes the value of 1 if the arc is set to be reconstructed at period $t \in \mathcal{T}$; it takes the value of 0, otherwise. Similarly, for every node $i \in \mathcal{N}'_k$ in network $k \in \mathcal{K}$, there is a binary variable Δw_{ikt} that takes the value of 1 if the node is set to be reconstructed at period $t \in \mathcal{T}$; it takes the value of 0, otherwise. Note that a component that is not destroyed is not necessarily functional due to network dependence to another component. For each arc $(i,j) \in \mathcal{A}'_k$ in network $k \in \mathcal{K}$ there is a binary variable y_{ijkt} that takes the value of 1 if the arc is functional at period $t \in \mathcal{T}$; it takes the value of 0, otherwise. Similarly, for every node $i \in \mathcal{N}'_k$ in network $k \in \mathcal{K}$, there is a binary variable w_{ikt} that takes the value of 1 if the node is functional at period $t \in \mathcal{T}$; it takes the value of 0, otherwise. In addition, for each space $s \in \mathcal{S}$ there is a binary variable z_{st} to denote if the space was prepared for at least one reconstruction process at period $t \in \mathcal{T}$, taking the value of 1 if the space is intervened; it takes the value of 0, otherwise. Also, we define the deviation variables δ^+_{iklt} and δ^-_{iklt} to indicate the surplus or deficit supply of commodity $l \in \mathcal{L}_k$ at node $i \in \mathcal{N}_k$ in network $k \in \mathcal{K}$ at period $t \in \mathcal{T}$. Based on the previous description of the variables and parameters involved, the proposed model for the INDP is represented as follows:

minimize

$$\sum_{t\in\mathcal{T}|t>0}\left(\sum_{s\in\mathcal{S}} g_{st}\Delta z_{st} + \sum_{k\in\mathcal{K}}\left(\sum_{(i,j)\in\mathcal{A}'_k} f_{ijkt}\Delta y_{ijkt} + \sum_{i\in\mathcal{N}'_k} q_{ikt}\Delta w_{ikt}\right)\right)$$
$$+\sum_{t\in\mathcal{T}}\sum_{k\in\mathcal{K}}\left(\sum_{l\in\mathcal{L}_k}\sum_{i\in\mathcal{N}_k}\left(M^+_{iklt}\delta^+_{iklt} + M^-_{iklt}\delta^-_{iklt}\right) + \sum_{l\in\mathcal{L}_k}\sum_{(i,j)\in\mathcal{A}_k} c_{ijklt}x_{ijklt}\right) \tag{2.1a}$$

subject to,

$$\sum_{j:(i,j)\in\mathcal{A}_k} x_{ijklt} - \sum_{j:(j,i)\in\mathcal{A}_k} x_{jiklt} = b_{iklt} - \delta^+_{iklt} + \delta^-_{iklt}, \quad \forall k\in\mathcal{K},\forall i\in\mathcal{N}_k,\forall l\in\mathcal{L}_k,\forall t\in\mathcal{T}, \tag{2.1b}$$

$$\sum_{l\in\mathcal{L}_k} x_{ijklt} \le u_{ijkt}w_{ikt}, \quad \forall k\in\mathcal{K},\forall(i,j)\in\mathcal{A}_k,\forall t\in\mathcal{T}, \tag{2.1c}$$

$$\sum_{l\in\mathcal{L}_k} x_{ijklt} \le u_{ijkt}w_{jkt}, \quad \forall k\in\mathcal{K},\forall(i,j)\in\mathcal{A}_k,\forall t\in\mathcal{T}, \tag{2.1d}$$

$$\sum_{l\in\mathcal{L}_k} x_{ijklt} \le u_{ijkt}y_{ijkt}, \quad \forall k\in\mathcal{K},\forall(i,j)\in\mathcal{A}'_k,\forall t\in\mathcal{T}, \tag{2.1e}$$

$$\sum_{i\in\mathcal{N}_k} w_{ikt}\gamma_{ijk\tilde{k}t} \ge w_{j\tilde{k}t}, \quad \forall k,\tilde{k}\in\mathcal{K},\forall j\in\mathcal{N}_{\tilde{k}},,\forall t\in\mathcal{T}, \tag{2.1f}$$

$$w_{ik0} = 0, \quad \forall k\in\mathcal{K},\forall i\in\mathcal{N}'_k, \tag{2.1g}$$

$$y_{ijk0} = 0, \quad \forall k\in\mathcal{K},\forall(i,j)\in\mathcal{A}'_k, \tag{2.1h}$$

$$w_{ikt} \le \sum_{\tilde{t}=1}^{t}\Delta w_{ik\tilde{t}}, \quad \forall k\in\mathcal{K},\forall i\in\mathcal{N}'_k,\forall t\in\mathcal{T}\mid t>0, \tag{2.1i}$$

$$y_{ijkt} \le \sum_{\tilde{t}=1}^{t}\Delta y_{ijk\tilde{t}}, \quad \forall k\in\mathcal{K},\forall(i,j)\in\mathcal{A}'_k,\forall t\in\mathcal{T}\mid t>0, \tag{2.1j}$$

$$\sum_{k\in\mathcal{K}}\left(\sum_{(i,j)\in\mathcal{A}'_k} h_{ijkrt}\Delta y_{ijkt} + \sum_{i\in\mathcal{N}'_k} p_{ikrt}\Delta w_{ikt}\right) \le v_{rt}, \quad \forall r\in\mathcal{R},\forall t\in\mathcal{T}\mid t>0, \tag{2.1k}$$

$$\Delta w_{ikt}\alpha_{ikst} \le \Delta z_{st}, \quad \forall k\in\mathcal{K},\forall i\in\mathcal{N}'_k,\forall s\in\mathcal{S},\forall t\in\mathcal{T}\mid t>0, \tag{2.1l}$$

$$\Delta y_{ijkt}\beta_{ijkst} \le \Delta z_{st}, \quad \forall k\in\mathcal{K},\forall(i,j)\in\mathcal{A}'_k,\forall s\in\mathcal{S},\forall t\in\mathcal{T}\mid t>0, \tag{2.1m}$$

$$w_{ikt}b_{iklt} \le b_{iklt} - \delta^-_{iklt}, \quad \forall k\in\mathcal{K},\forall i\in\mathcal{N}^*_k,\forall l\in\mathcal{L}_k,\forall t\in\mathcal{T}, \tag{2.1n}$$

$$\delta^+_{iklt} \ge 0, \quad \forall k\in\mathcal{K},\forall i\in\mathcal{N}_k,\forall l\in\mathcal{L}_k,\forall t\in\mathcal{T}, \tag{2.1o}$$

$$\delta^-_{iklt} \ge 0, \quad \forall k\in\mathcal{K},\forall i\in\mathcal{N}_k,\forall l\in\mathcal{L}_k,\forall t\in\mathcal{T}, \tag{2.1p}$$

$$x_{ijklt} \ge 0, \quad \forall k\in\mathcal{K}, \quad \forall(i,j)\in\mathcal{A}_k,\forall l\in\mathcal{L}_k,\forall t\in\mathcal{T}, \tag{2.1q}$$

$$w_{ikt} \in \{0,1\}, \quad \forall k \in \mathcal{K}, \forall i \in \mathcal{N}_k, \forall t \in \mathcal{T} \tag{2.1r}$$
$$y_{ijkt} \in \{0,1\}, \quad \forall k \in \mathcal{K}, \forall (i,j) \in \mathcal{A}'_k, \forall t \in \mathcal{T}, \tag{2.1s}$$
$$\Delta w_{ikt} \in \{0,1\}, \quad \forall k \in \mathcal{K}, \forall i \in \mathcal{N}'_k, \forall t \in \mathcal{T} \mid t > 0 \tag{2.1t}$$
$$\Delta y_{ijkt} \in \{0,1\}, \quad \forall k \in \mathcal{K}, \forall (i,j) \in \mathcal{A}'_k, \forall t \in \mathcal{T} \mid t > 0, \tag{2.1u}$$
$$\Delta z_{st} \in \{0,1\}, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T} \mid t > 0. \tag{2.1v}$$

The objective function (2.1a) is composed by five different terms that are minimized simultaneously. The first term is the cost of preparing the geographical spaces for the recovery (drilling, excavations, etc.), the second and third term are the reconstruction costs associated with the links and the nodes respectively, the fourth term is the cost associated with the under or over supply of the given demands, and the fifth term is the total cost flow in the system. Constraints (2.1b) guarantee the flow balance in each of the networks, enforcing the correct net flow for each node in the system, considering its demands and its under or over supply. Constraints (2.1c), (2.1d), and (2.1e) guarantee that there can be flow trough each arc only if its starting node, its ending node, and the arc itself are functional. Also, these constraints ensure that the total flow trough each arc is under its maximum capacity. Constraints (2.1f) model the physical and logical interdependence between nodes, guaranteeing that nodes are functional only if their parent nodes are also functional. Constraints (2.1g) and (2.1h) enforce that the initially damaged nodes and arcs (respectively) are not functional right after the destructive event (time $t = 0$). Constraints (2.1i) and (2.1j) guarantee that each initially damaged node or arc can become functional only after being fully recovered. Constraints (2.1k) model the use of limited resources, guaranteeing that the amount used of each resource type never surpasses its availability, at any time. Constraints (2.1l) and (2.1m) enforce that at every time a node or an arc is repaired, the geographical spaces that contain them must be prepared. Constraints (2.1n) guarantee that, for the specified set of nodes that require it, each node can be functional only when covering its full demands. Finally, constraints (2.1o) - (2.1v) describe the type of each decision variable, being either binary or simply non-negative.

Note that the proposed td-INDP enforces integrality constraints for the variables associated with the recovery and the functionality of each element, since these are binary variables that indicate if a given element is recovered or is functional at each recovery period. However, td-INDP allows relaxing such constraints in case the modeled system permits partial functionality states or partial recovery of a damaged element. If such a case is assumed, the proposed td-INDP could be used to model a recovery process where each element has a different speed of recovery, that depends on how much resources are devoted to it. Then, using constraints (2.1k), one could easily consider different recovery speeds, by modeling the percentage in which each element is recovered at each period. In this case, parameters h_{ijkrt} and p_{ikrt} would indicate how much of resource r is required to recover 100% of each given link and node. Moreover, if one defines a resource associated with 'recovery time', these parameters could directly indicate how much time is required to recovery each element.

2.3 Illustrative examples

The proposed td-INDP formulation is intended to allow analyzing a broad range of systems of networks. It permits having one or multiple commodities flowing through one or more networks simultaneously, while considering multiple types of resources simultaneously. To exemplify the capabilities and versatility of the proposed td-INDP model, we present two different realistic case studies. First, we study the recovery process associated with the Colombian road network, after being damaged by an earthquake. This case shows how the proposed model can be used to optimize the recovery process of a given infrastructure network, while considering multiple commodities flowing through it. Second, we study the recovery of the water, power, and gas networks in Shelby County, after the occurrence of a simulated earthquake. This example shows how the td-INDP model can optimize the recovery process of multiple interdependent infrastructure networks.

2.3.1 Case study 1: Colombian road network

This example illustrates the use of the td-INDP model on a single-network multi-commodity system. In this example, we study the recovery process associated with a simulated disaster scenario consistent with an earthquake that occurred in Colombia on January 25th, 1999, with magnitude $M_w = 6.1$ and an epicenter located at $4.41°$ N, $-75.72°$ W. Such an earthquake had a tremendous negative impact on Colombia, but in particular in the cities of Pereira and Armenia, causing 1200 deaths, more than 250 000 people affected, and more than US$1.2 billion in economic losses (Yoshimura and Croston, 1999; Sánchez-Silva et al., 2000).

This case is of relevance, since even though the Colombian road system is studied as an isolated network, the commodities that flow through it are very numerous. To model the inter-city Colombian road system, we considered the 54 largest and most important cities. To connect them, there are 60 main roads that can be used for freight and human transportation. Figure 2.1 shows the primary road transportation network along with its graph representation. For each of these cities, we considered the demand/supply of 271 different types of products, such as flowers, boxes, paint, sugar, flour, etc., accounting for more that 169 600 000 tons of cargo being transported through the system.

Figure 2.2 shows four different stages of the Colombian road network destruction and reconstruction process. First, Figure 2.2a represents the network before the earthquake. Second, Figure 2.2b shows the damaged network after the earthquake. Then, Figure 2.2c and Figure 2.2d show the reconstruction process after four and

15 recovery periods, respectively. For this particular network, the amount of arcs repaired by period –modeled with constraints (2.1k)– is limited to 1, so as to illustrate a gradual step-by-step reconstruction process. As expected, the td-INDP gave high priority in the early stages to reconnecting nodes that were totally disconnected from the network after the earthquake, but after having reconnected the whole system it focused on reducing operational costs.

2.3.2 Case study 2: Utilities in Shelby County, TN.

This example illustrates some of the td-INDP capabilities regarding the modeling of multiple interdependent infrastructure networks and designing recovery strategies for all of them simultaneously. In particular, this case study analyses streamlined versions of the power, water, and gas networks in Shelby County, TN, USA. Shelby County is known for containing the city of Memphis, and also for being at risk from earthquakes due to the New Madrid Seismic Zone (NMSZ). The test network descriptions used for this study were adapted from Hernandez-Fajardo and Dueñas-Osorio (2010), Hernandez-Fajardo and Dueñas-Osorio (2013), and Song and Ok (2010). Regarding the disaster realization, Adachi and Ellingwood (2009) presented a realistic earthquake scenario for Shelby County, with epicenter at 35.3° N and 90.3° W (33 km from Memphis center). For this example, we detail the recovery process associated with a disaster realization consistent with an earthquake with magnitude $M_w = 8$.

In this case study, there are two main interdependencies of interest. First, there is a physical interdependence between the power networks and the gas and water networks, given that for a pumping station to be functional, it is required that at least one power station (from a set of covering stations) is functional as well. Second, this case study takes under consideration the geographical interdependence from co-location between the water and the gas networks. Given that both networks are underground, there is a

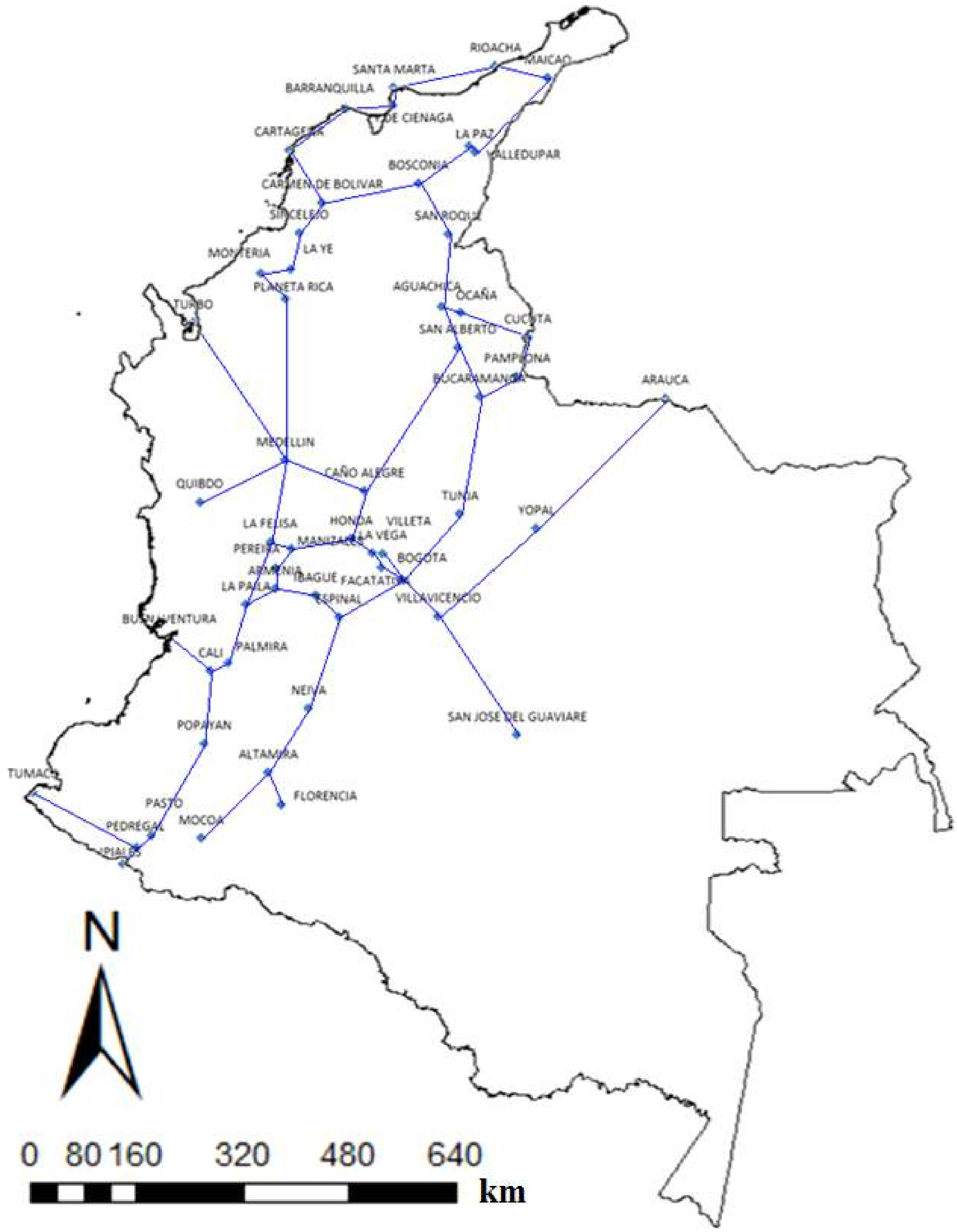
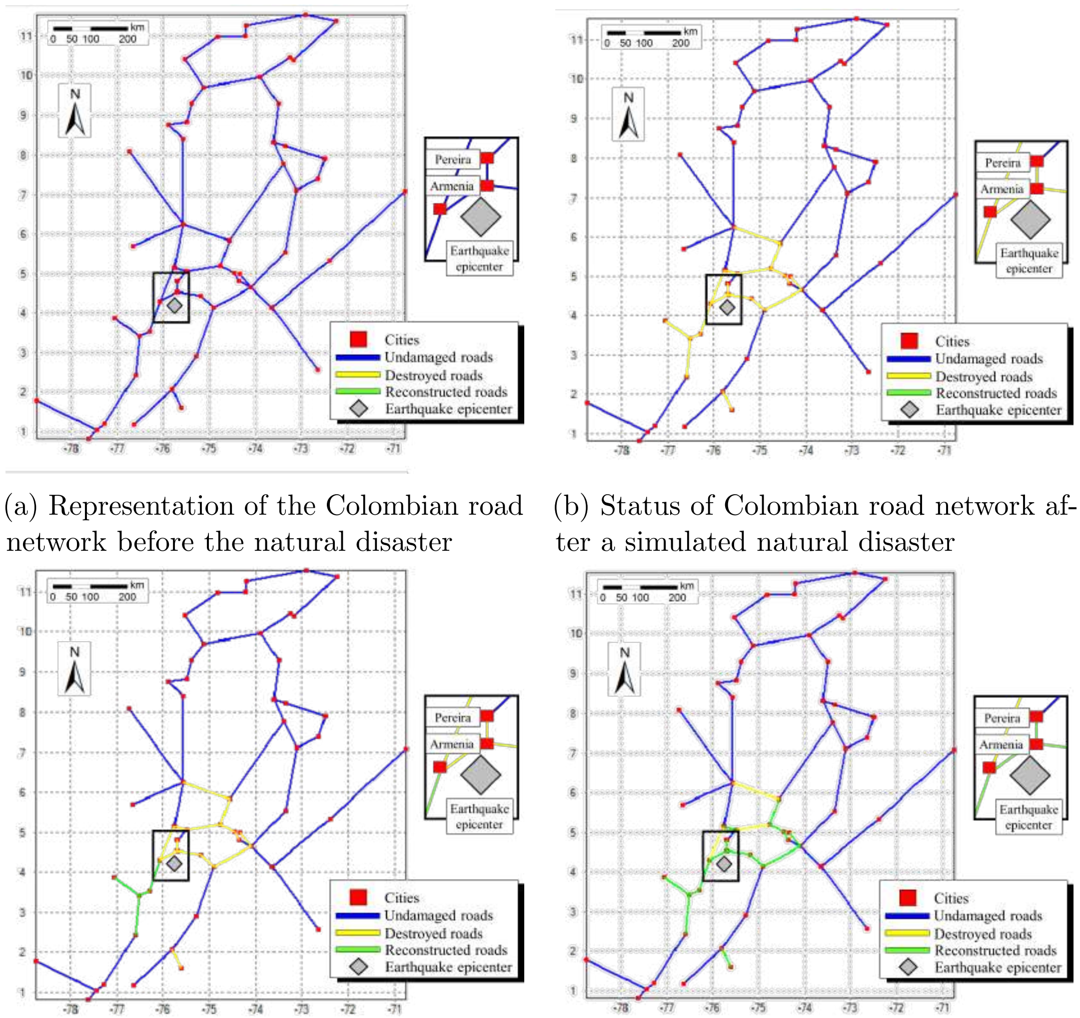


Figure 2.1 : Colombian primary road network



(a) Representation of the Colombian road network before the natural disaster

(b) Status of Colombian road network after a simulated natural disaster

(c) Status of the Colombian road network after 4 periods of the td-INDP

(d) Status of the Colombian road network after 15 periods of the td-INDP

Figure 2.2 : Graphical representation of the td-INDP iterative reconstruction of the primary Colombian road network after a simulated natural disaster (earthquake)

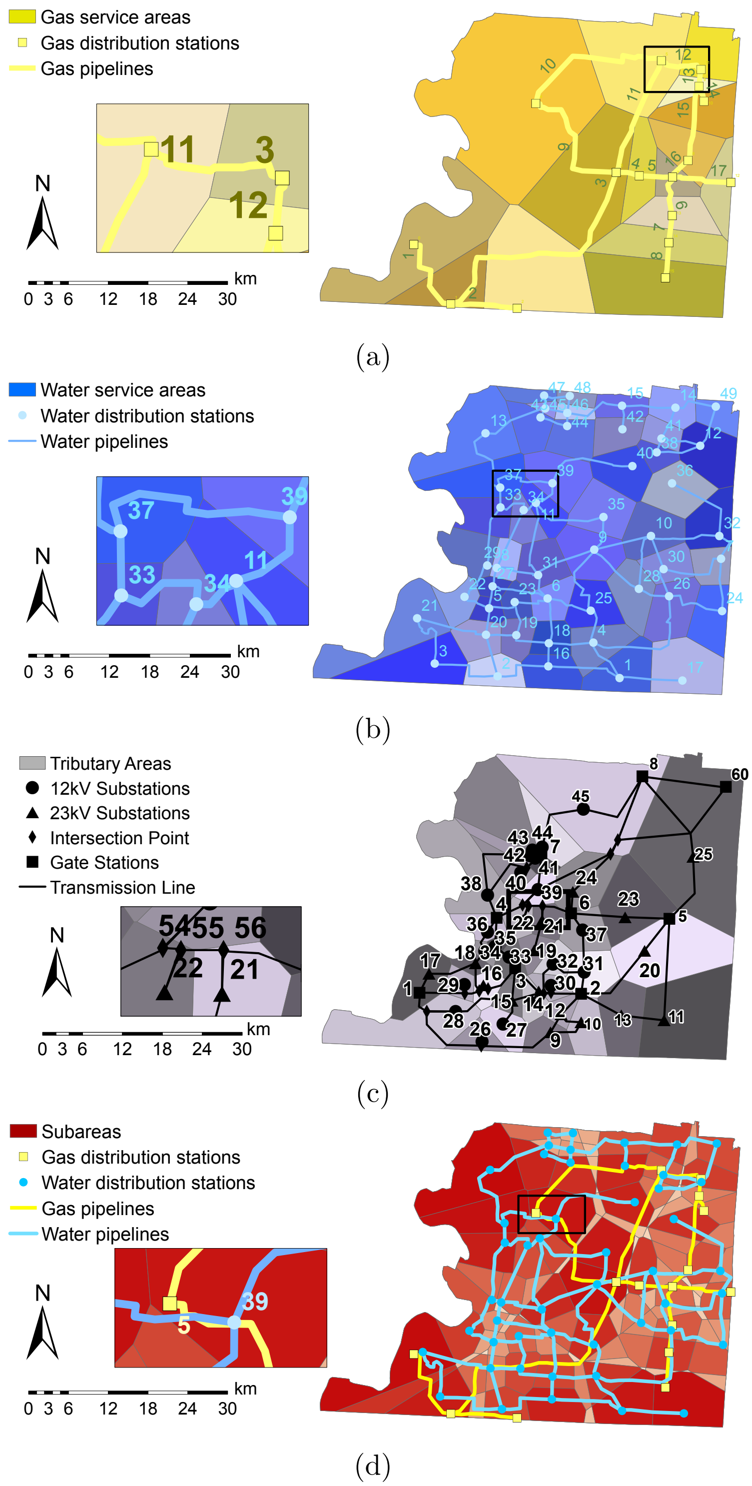


Figure 2.3 : Critical infrastructure networks in Shelby County, TN. (González et al., 2016b)

shared-area preparation cost related to the reconstruction process of each component, that is, there is a saving potential by repairing co-located components from the water and the gas networks simultaneously. Note that even though some previous works on interdependent infrastructure recovery (Lee II et al., 2007; Cavdaroglu et al., 2011; Nurre et al., 2012; Sharkey et al., 2015) have mentioned the existence and importance of geographical correlation, most have focused on the failure correlations between components, but not in the savings in time, money, and efforts that could be made if assigning simultaneous recovery jobs for co-located components. Given that this example considers geographical interdependence, it is important to define a set of geographical spaces $\mathcal{S}$. In this case, $\mathcal{S}$ is defined as the set of areas resulting from intersecting the service areas of the gas and water networks, which are the ones under geographical interdependence. Figure 2.3 shows the gas, water, and power networks, as well as the intersection areas that constitute $\mathcal{S}$. Note that these intersection areas are mutually exclusive and collectively exhaustive. The td-INDP model can now be used to find the optimal recovery strategy for the system, which in this case consists of multiple interdependent infrastructure networks.

The amount of elements that can be repaired are limited to 3 per period in this case study, so as to enable the recovery of one element per network each time. To illustrate the recovery process associated with the disaster scenario studied, Figure 2.4 shows the performance evolution associated with the damage and subsequent recovery of the studied water, power, and gas networks. For this study, the average performances associated with each network are shown in Table 2.1. These performances were defined as the average percentage of demand that is supplied, for each network, during the recovery process. These can be determined using the fourth term of the objective function (2.1a) of the td-INDP, if the under-supply costs for each commodity are assumed to be equal, and the over-supply costs are zero.

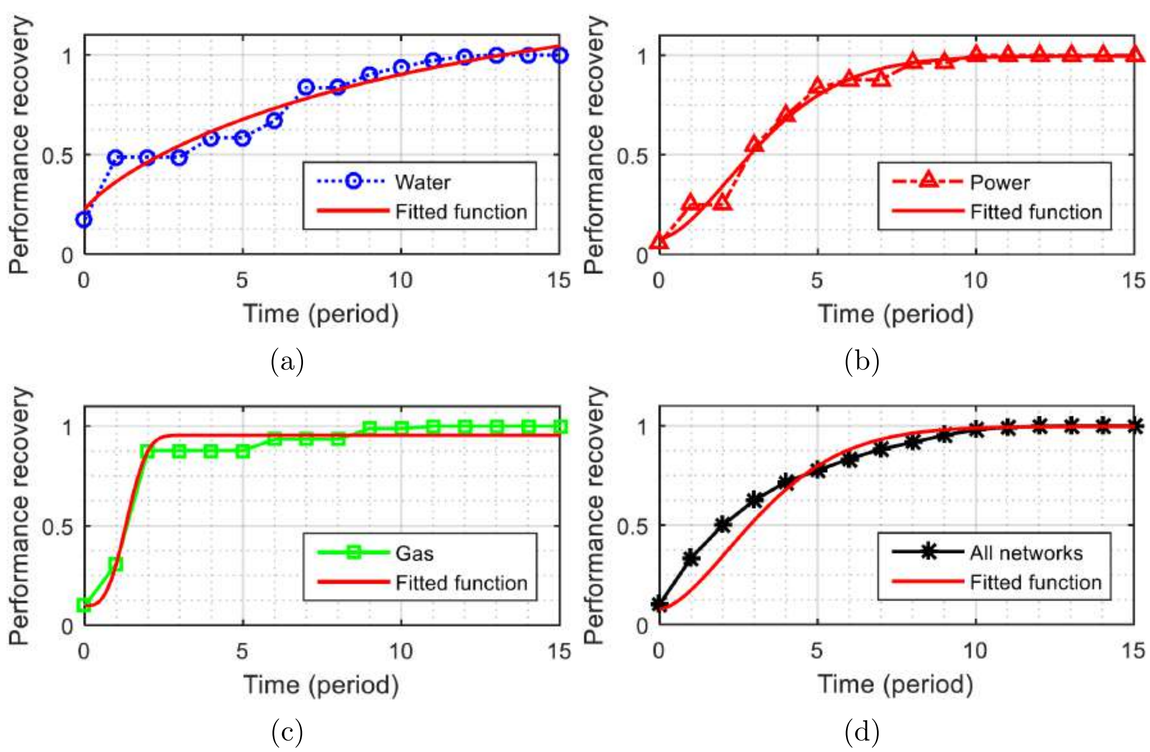


Figure 2.4 : Optimal performance recovery for the (a) water, (b) power, and (c) gas networks, as well as (d) the combined system of infrastructure networks in Shelby County, TN., after a disaster consistent with an earthquake of magnitude $M_w = 8$ (González et al., 2016a)

Figure 2.4d shows that the optimal performance recovery of the full system tends to be smooth, relative to the recovery of each of the individual networks. This shows that the recovery that optimizes the overall resilience of the system, does not necessarily represent the strategy that optimizes the resilience of each individual network. In fact, it can be seen that the scarce resources imply that, at some periods, while some networks are recovered others have to wait with their performance unchanged. For example, all three networks experience performance recovery during the first period, but at the second period the gas network seems to have a priority over the others, as it is recovered to almost full performance while the other two networks were not repaired at all. Then, at period 3, only the power network was recovered, while the others remain unchanged. This behavior, observed during the whole recovery process, shows that in order to optimize the resilience of a system of interdependent networks, it is important to consider all networks simultaneously, as opposed to optimizing for each of them independently. Table 2.1 shows that the gas network indeed had priority over the other networks, as its average performance was greater than the other two networks. This can be explained by the fact that the gas network has much less redundancies compared to the other networks, thus each time an element is repaired on this network, there is a greater impact on its recovery compared to the other networks.

Table 2.1 : Average performance of the optimal recovery strategies found (González et al., 2016a)

	Infrastructure system			
	Water	Power	Gas	Full system
Av. Performance	0.747	0.770	0.856	0.788

Also, to showcase another possible application of the td-INDP, we used the obtained optimal recovery strategies to fit a continuously-differentiable function for each network

–of the form described in equation (2.2)–, so as to provide useful benchmarks that can be used by other researchers –such as for gradient descent algorithms and other methods that are based on such continuously-differentiable functions. Table 2.2 describes the coefficients for the optimally-fitted functions for each performance recovery, as well as their associated coefficients of determination (R^2).

$$\hat{P}(X) = \hat{A} - \hat{B}e^{\hat{E}-\hat{C}X^{\hat{D}}} \tag{2.2}$$

Table 2.2 : Coefficients of fitted recovery functions (González et al., 2016a)

	Infrastructure system			
Coeff.	Water	Power	Gas	Full system
$\hat{A}$	1.38	0.996	0.955	0.996
$\hat{B}$	0.65	0.862	0.832	0.917
$\hat{C}$	0.133	0.11	0.286	0.111
$\hat{D}$	0.825	1.639	3.063	1.633
$\hat{E}$	0.58	0.062	0.03	0.001
R^2	0.954	0.986	0.97	0.986

Such utilization of the td-INDP solutions demonstrates that the proposed optimization model not only can be used to inform decision-makers about the optimal recovery strategies for a damaged system of interdependent networks, but also to provide benchmarks for other academic and practical studies.

An analogous application of the td-INDP is detailed in Section 5.2, where the td-INDP is used to provide benchmark failure/recovery time series, to feed input data to a system identification framework, which focuses on extracting and recreating key features of the recovery dynamics associated with interdependent networks.

2.4 Conclusions

This Chapter describes a new model for defining the optimal reconstruction strategy for a partially destroyed system of interconnected infrastructure networks, that accounts for functional interdependencies between the associated networks while incorporating for the first time reconstruction savings due to co-located components that could be repaired simultaneously.

First, we introduce the Interdependent Network Design Problem (INDP), which seeks the minimum-cost reconstruction strategy, subject to budget, resources, and operational constraints. Then, we propose a Mixed Integer Programming (MIP) mathematical formulation denominated time-dependent INDP (td-INDP) model, that extends the ideas of the widely known Network Design Problem (NDP) (Johnson et al. (1978); Ahuja et al. (1993); Hu (2012)) to include interdependencies and other operational constraints.

The proposed td-INDP model determines the optimal reconstruction strategy of a damaged system of interdependent infrastructure networks, seeking the least-cost recovery configuration while accounting for constraints related to the existence of limited resources (such as budget, time, manpower, etc.), flow capacities, demand/supply of commodities, and functional relations between the networks, among others.

Even though the proposed td-INDP is defined as a post-event tool, this mathematical model has the potential to also be used for pre-event assessments associated with prevention and mitigation, to reduce the negative impact of natural and targeted damaging events. Such a generalization can be achieved by extending the td-INDP mathematical model to include uncertainty, particularly associated with the failure modes of the system. To this end, Section 3.1 will introduce a simulation-based approach that allows analyzing the expected recovery dynamics of a given system of interdependent networks. This idea will be furthered discussed in Chapter 4, where we address multiple sources of uncertainty, and show how the td-INDP can integrate them in its analysis.

However, in order to facilitate using the td-INDP in simulation-based studies, it is imperative that we understand the computational complexity associated with it, and particularly, that we have tools to solve it in a time-efficient manner. To this end, Chapter 3 presents a detailed study on the structure and complexity of the td-INDP, and proposes heuristic and analytical approaches to enhance its solution times.

Chapter 3

Understanding and Improving the Efficiency of the td-INDP

In Section 2.2, we described how the proposed td-INDP model can be used to quantify the effects of a disaster over the performance of a system of interdependent networks. More over, we showed how the td-INDP model can be used to determine the optimal recovery strategy for a damaged system, such that the associated recovery costs are minimized –or more specifically, such that the average performance is maximized–, offering an invaluable tool for emergency-response and planning situations. However, it is imperative to ensure that the td-INDP model can be solved in a time-efficient manner, such that it can be used in realistic time-critical applications*.

Moreover, as hinted at the end of the previous Chapter, the proposed td-INDP model could be used for purposes beyond emergency response, such as mitigation and preparedness, providing stakeholders useful information regarding the system vulnerability. In particular, the td-INDP model could be used to perform probabilistic studies on a given system, to detect critical elements, forecast recovery costs and resource utilization, and estimate recovery times, among others. Nevertheless, these probabilistic studies often require considering the effects of multiple damage scenarios, thus increasing the computational complexity of the associated optimization processes.

Given this, it is imperative to understand and improve the computational efficiency

*This Chapter is based on the ideas and contents presented in González et al. (2016b) and González et al. (2015). The definitive versions of these papers are available at www.onlinelibrary.wiley.com and open.library.ubc.ca of the proposed td-INDP model. To achieve this, Section 3.1 presents a flexible heuristic algorithm to solve the td-INDP. This heuristic approach is based on dividing the set of recovery periods (which determines the recovery time horizon) into smaller sets, such that the td-INDP can be solved systematically using these reduced time sets on a running horizon scheme. Then, Section 3.2 presents an analytically-oriented study on the properties and the structure of the td-INDP model. In particular, we start by determining how diverse topological properties of the studied systems influence the td-INDP solving times, performing a detailed sensitivity analysis of the model parameters. Then, we describe the particular structure of the td-INDP model, and show how this structure, which reflects a block-decomposable system, could be exploited to enhance its solving speeds and size capabilities.

3.1 Heuristic approach

In Section 2.3.2, we showed how the td-INDP can be used to calculate the optimal recovery strategy of the water, power, and gas networks in Shelby County, TN. associated with an earthquake disaster scenario. However, in order to determine the optimal recovery strategy, the td-INDP model requires that the recovery planning horizon is given, as it is modeled by a set $\mathcal{T}$ that is fixed during the optimization process. By looking closely at the td-INDP formulation, we can observe that both the number of terms in objective function (2.1a) and the number of constraints (2.1b)-(2.1v) increase linearly as a function of the cardinality of $\mathcal{T}$. Thus, it is to be expected that the number of recovery periods considered by the td-INDP will have a relevant impact on its time-complexity.

To illustrate the relation between the cardinality of $\mathcal{T}$ and the solution times associated with the td-INDP, we randomly generated 100 different damage realizations

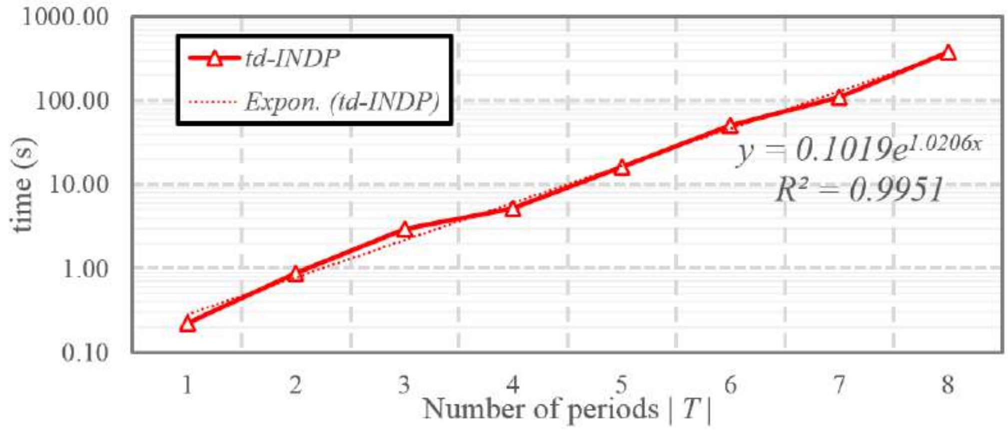


Figure 3.1 : Average solving times for the proposed td-INDP model

consistent with a magnitude $M_w = 9$ earthquake and the realistic system of networks described in Section 2.3.2, and measured their average solution times, while considering multiple planning horizons. We selected this large earthquake magnitude, in order to test the td-INDP efficiency in worst-case scenario conditions, related to highly damaged systems. Figure 3.1 shows the average td-INDP solution times related to the aforementioned disaster realizations, considering planning horizons that spanned from 1 to 8 periods. As expected, the average td-INDP solving times highly increased as a function of $|\mathcal{T}|$. In particular, Figure 3.1 shows that the average solution times increase exponentially with the size of the planning horizon. This exponential increase would imply that, independently of the level of destruction and the size of the studied network, there will be always a planning horizon limit, for which the td-INDP could not be fully solved in a realistic time range. This property evidences the importance of developing solution approaches to the td-INDP model that are scalable to large planning horizons. In order to obtain efficient recovery strategies for scenarios defined over long planning horizons, we propose a heuristic approach that improves the solving time of the td-INDP by solving it using a piece-wise strategy. Such time-decomposition approaches have been shown to be particularly successful for scheduling and routing problems that involve planning for multiple periods in the future (Campbell, 2008), but now we explore their suitability in recovery planning applications. The proposed heuristic approach, denominated the iterative INDP (iINDP), is detailed as follows.

3.1.1 The iterative INDP (iINDP)

Instead of solving the td-INDP for all periods $t \in \mathcal{T}$ at once, we propose splitting $\mathcal{T}$ into multiple smaller recovery periods, to then use the td-INDP to systematically solve the recovery process associated with each of these reduced planning horizons.

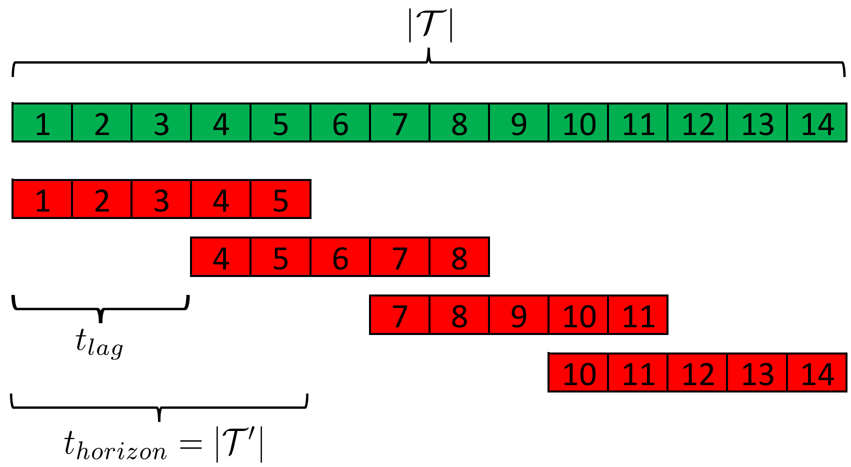
Assume that each reduced planning horizon is of size $t_{horizon} \leq |\mathcal{T}|$. Then, the proposed algorithm starts by solving the td-INDP model focusing only on periods 0 to $t_{horizon}$ (even though it may not be optimal for all periods $t \in \mathcal{T}$). The obtained recovery strategy is guaranteed to be the optimal for periods $\{0, 1, ..., t_{horizon}\}$, minimizing the average recovery costs for these periods. Then, considering the recovery strategy proposed for periods $\{0, 1, ..., t_{horizon}\}$, we update all sets associated with the damaged nodes and arcs, $\mathcal{N}'$ and $\mathcal{A}'$, respectively. Then, based on the updated sets of damaged elements, and using time $t_{horizon} + 1$ as the initial period, use the td-INDP again to find the optimal recovery strategy for periods $t_{horizon} + 1$ to $2t_{horizon}$. Using the new recovery information for periods $\{t_{horizon} + 1, t_{horizon} + 2, ..., 2t_{horizon}\}$, we update again sets $\mathcal{N}'$ and $\mathcal{A}'$, and repeat the same process until a recovery strategy for the full planning horizon is obtained.

However, note that in some cases splitting the original time horizon into multiple disjoint periods may not offer an accurate approximation of the optimal recovery strategy for the whole time horizon studied. The quality of this approach will depend on multiple factors, such as the level of interdependency between the studied networks, how limited the resources are, or how large the initial destruction is, among others.

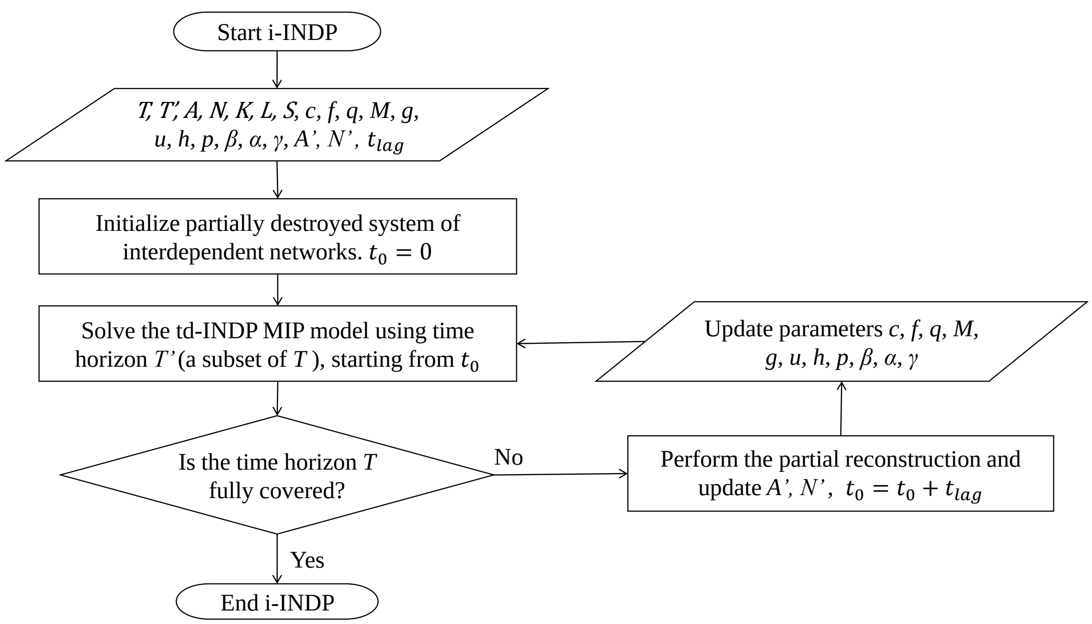
To circumvent this issue and allow 'communication' between the different reduced planning horizons, we should allow certain overlapping between consecutive time horizons. By including such overlaps, we may increase the quality of the overall solutions, by letting the optimizer update the recovery plan for the final periods of one reduced horizon, using information of the following reduced horizon. To model this, define an additional parameter t_{lag}, to indicate how many periods are being advanced at each iteration in which the td-INDP is solved, such that $t_{lag} \leq t_{horizon}$.

Considering these two parameters, the algorithm starts by solving the td-INDP model using as time horizon the periods 0 to $t_{horizon}$. Then, the proposed recovery strategy from period 0 to t_{lag} is saved, and the sets of destroyed nodes and destroyed arcs are updated according to the saved recovery strategy (taking into account which elements are repaired at each period). Then, for the next iteration, the starting and ending periods will shift in t_{lag} periods with respect to the previous ones. This algorithm, denominated the iterative INDP (iINDP), iteratively uses the td-INDP model until the original time horizon has been fully explored. In order to illustrate the proposed algorithm, Figure 3.2 depicts the proposed decomposition of the time horizon and presents a diagram that describes the iINDP. A more detailed description of the iINDP is shown in Algorithm 1.

To illustrate the time savings involved in adopting the iINDP, Figure 3.3 shows a comparison between the average solution times using the td-INDP and the iINDP, for the scenarios previously depicted in Figure 3.1. For this case we used $t_{lag} = 0$ and $t_{horizon} = 1$, which is expected to be the most time-efficient iINDP configuration. Figure 3.3 shows that, while the td-INDP times increase exponentially with the cardinality of $\mathcal{T}$, the iINDP times only increase linearly. This results in an average speedup of more than 20 times when solving for 5 periods, and more than 300 times when solving for 8 periods.



(a) Graphical depiction of the difference between the INDP model (green) and the iINDP (red)



(b) iINDP flow chart

Figure 3.2 : Graphical representation of the (a) time-horizon decomposition, and (b) illustrative flow chart for the iINDP algorithm

Algorithm 1 Iterative Interdependent Network Design Problem (iINDP)

Input: $\mathcal{T}$, $t_{horizon}$, t_{lag}, $\mathcal{A}$, $\mathcal{N}$, $\mathcal{K}$, $\mathcal{L}$, $\mathcal{S}$, $\mathcal{R}$, $\mathcal{A}'$, $\mathcal{N}'$, c, f, q, M, g, u, h, p ,b, v, α, β, γ
Output: $\hat{x}, \hat{y}, \hat{z}, \hat{w}, \Delta\hat{y}, \Delta\hat{w}$

```
procedure iINDP
    t_0 ← 0
    while t_0 ≤ |T| do
        Solve the td-INDP using periods t_0 to t_horizon
        for all ∀t ∈ {t_0...t_0 + t_lag}  do
            x̂, ŷ, ŵ, ẑ, Δŷ, Δŵ ← x, y, w, z, Δy, Δw
        end for
        A' ← A'\{(i, j) : ŷ_ijklt = 1, ∀t ∈ t_0...t_0 + t_lag}
        N' ← N'\{i : ŵ_iklt = 1, ∀t ∈ t_0...t_0 + t_lag}
        update c, f, q, M, g, u, h, p ,b, v, α, β, γ        ▷ External changes
        t_0 ← t_0 + t_lag
    end while
    return x̂, ŷ, ŵ, ẑ, Δŷ, Δŵ
end procedure
```

Even though the iINDP does not theoretically guarantee the optimality of the obtained solution, it is expected to return good solutions for realistic applications. For example, by inspecting the td-INDP performance recovery described in Figure 2.4d, we can see that the system's performance evolves with a monotonically decreasing non-negative slope, i.e., the performance is always increasing, but the rate at which it does reduces with time. Note that this property is observed only for the full system, but not for the performance recovery associated with each individual network, where the recovery processes have multiple points in which the rates of recovery oscillate, and even stagnate.

Considering that our main objective is associated with optimizing the resilience of the full system of interdependent networks, it is desirable for the iINDP that the slope of the optimal performance is always non-negative and that it decreases monotonically. This can be said since the iINDP is based on finding the recovery strategy that maximizes the performance on the immediate subsequent periods, which is analogous to finding the strategy that maximizes the slope at each time step. Thus, in cases where the optimal performance recovery is smooth, it is likely that the iINDP will return a strategy that approximates the behavior of the optimal one.

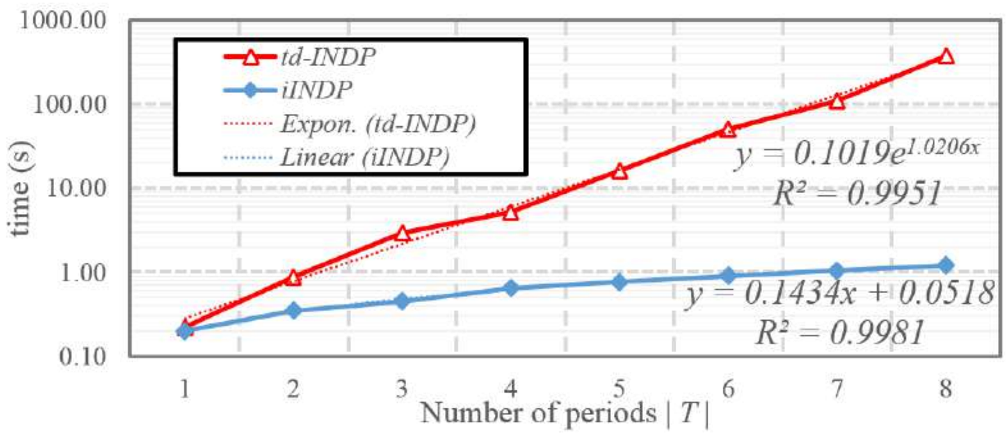


Figure 3.3 : Solving time scalability comparison between the proposed INDP MIP model (td-INDP), and the iterative INDP (iINDP)

To illustrate the quality of the iINDP solutions with respect to the td-INDP and other heuristic approaches, Figure 3.4 shows the performance recovery for the system of water, gas, and power networks in Shelby County, TN., associated with a disaster realization consistent with an earthquake of magnitude $M_w = 8$. This Figure shows the performance that would result after following the recovery strategies proposed by the td-INDP, the iINDP, and two other heuristics based on Variable Neighborhood Search (VNS) and permutations. It can be seen that the iINDP solution results in a quasi-optimal recovery, only 0.49% from the optimal solution (despite having used $t_{lag} = 0$ and $t_{horizon} = 1$), whereas the other heuristics achieved solutions 8.27% and 18.60% from the optimal. Since the optimal performance recovery for this disaster realization depicted the desired features (non-negative slope that decreases monotonically), the iINDP was expected to return a good solution. However, for systems in which the performance recovery has multiple inflection points, the iINDP may not return good quality solutions if a small $t_{horizon}$ is used, since the algorithm may not have adequate information to account for the effects that decisions at close periods will have in the long term.

Moreover, even if the shape of the optimal recovery strategy is 'ideal', the iINDP solutions may loose quality for planning horizons that are too long, since the effects of taking sub-optimal decisions will accumulate at each iteration. Figure 3.5 illustrates this effect, when using the iINDP (with $t_{lag} = 0$ and $t_{horizon} = 1$) to recover a fully disconnected power network composed of 1000 nodes, and recovering only one link per iteration. This Figure shows the connectivity of the network (measured by the ratio between the number of disconnected components –labeled as $|C|$– and the number of nodes in the network –labeled as N–), using the iINDP and three other heuristics that are based on selecting links from a predefined pool of links (of sizes 1, 2, and 1 –labeled as M–). For this example, time $t = 1$ refers to the approximate total time required to gain full connectivity, which corresponds to approximately 1000 recovery periods. It can be seen that even though the iINDP reconnects the system in a relatively efficient manner, the negative effects associated with the non-optimal decisions taken at each recovery period accumulate in time.

Furthermore, given that each iteration of the iINDP is based on solving a separate optimization problem, the iINDP allows for parameter updating, as opposed to the td-INDP. This makes the iINDP a more flexible tool when studying problems where parameters such as costs, resources, or capacities are difficult to forecast well in advance. Moreover, the iINDP also allows updating all the associated sets at each iteration, enabling modeling aftershock scenarios where there are multiple subsequent destructive events. Additionally, note that parameters t_{lag} and $t_{horizon}$ could also vary at each iINDP iteration, such that they can adapt to the desired level of resolution

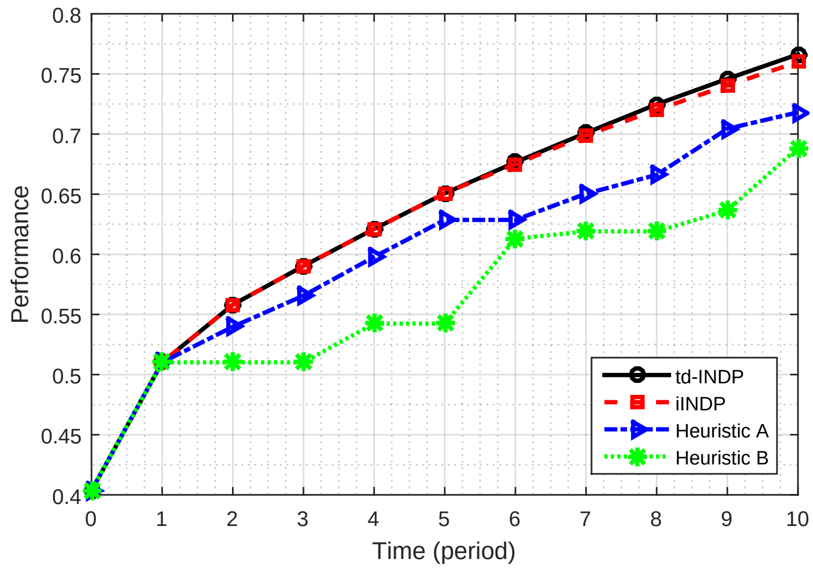


Figure 3.4 : Performance recovery for the system of infrastructure networks in Shelby County, TN., after a disaster consistent with an earthquake of magnitude $M_w = 8$, using the td-INDP, the iINDP, and two permutation-based heuristics

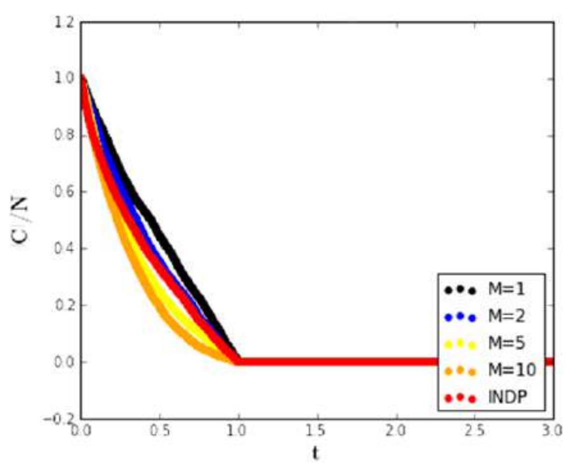


Figure 3.5 : Recovery of artificial power grid using the iINDP and a guided percolation-based heuristic (Smith et al., 2017)

and quality required at each recovery period. For example, one may give priority to the quality of the recovery strategy at initial stages with respect to periods far from the current period. In such a case, for the initial iterations one could choose relatively large $t_{horizon}$ and small t_{lag} values, whereas for the final recovery periods one could use relatively small $t_{horizon}$ values. These time-adaptability features are desirable to the end user of the iINDP, since he could adapt the algorithm to the desired quality level, while considering time constraints in the solution times.

3.1.2 Illustrative example

As previously mentioned, one of the advantages of using the proposed iINDP is that it enables performing simulation-based studies, thanks to the considerable associated speedups. For this illustrative example, we perform a simulation study to estimate the average costs and performance of the water, power, and gas networks in Shelby County, TN., subject to diverse disaster scenarios. The simulated disaster realizations are constructed following the propagation curves and other methodologies described by Adachi and Ellingwood (2009), who presented a realistic earthquake scenario for Shelby County, with an approximate average magnitude of $M_w = 7.5$. Considering this magnitude, this study includes magnitudes within a range of $M_w \in \{6, 7, 8, 9\}$. For each of such magnitudes, the number of Monte-Carlo replications is limited to 1000, such that the results show steady behaviors.

For this example, we set different maximum amounts of components to be repaired per iteration of the iINDP, in order to analyze the sensitivity of the solutions with respect to these amounts. This number, denoted by v – as it is modeled using constraints (2.1k)– spans from 3 to 12 components. We selected 3 components as the lower bound of v, such that it is always feasible to reconstruct at least one component from each of the three networks. Likewise, we selected 12 as the upper bound of

v, given that for greater values, constraints (2.1k) show a limited impact on the reconstruction strategies.

Figures 3.6-3.9 show the results associated with the evolution of the costs involved in the recovery process; as expected, note that the costs depend directly on the magnitude of the earthquake. The cost evolution analysis is presented for the four different values of the maximum number of reconstructed components per iteration ($v = 3, 6, 9, 12$) and for each of the four magnitudes ($M_w = 6, 7, 8, 9$). We normalized the cost values using as reference point the cost of reconstructing the complete system, as we focus on studying the cost behavior, and not on their absolute values.

Figure 3.6 shows the evolution of the flow costs as a fraction of the minimum feasible flow cost associated with a fully recovered network, i.e., the values in this Figure correspond to the ratio between the actual flow cost and the minimum flow cost related to a fully operational system (which would indicate the optimal configuration). Note that for low-magnitude scenarios, such costs converge to the optimal, but for high-magnitude scenarios, the costs converge to values up to 33.1% greater than the optimal flow cost. This suggests that for high-magnitude earthquake events (where a large number of components is destroyed), at the final stages of the reconstruction the remaining feasible recovery jobs have a high cost, hence the savings in flow costs would be less than the actual costs of reconstructing additional components. This is seen in real life contexts, where some components cannot be repaired immediately due to cost limitations. Nevertheless, posterior reduction in the operative and reconstructions costs could enable repairing the remaining components as well. Note that a network flows approach with an explicit time index (such as the td-INDP) would not be able to adapt its costs to the current state of the network, but this can be easily carried out by using the iINDP. Given that in the initial stages, where few reconstructions have been performed, the flow costs are mostly an indicator of how much of the demanded commodities are being supplied, where low initial flow costs (below 1) indicate that there is an average lack-of-supply issue in the system (i.e., the flows are not enough to cover the whole demand in the system). This is supported by the fact that higher earthquake magnitudes have associated lower initial flow costs. Also, in general the final flow costs do not seem to depend strongly on the parameter v, where the highest difference between the final costs for a particular magnitude is only 5.3%. This highlights the robustness of the iINDP and its results, since independently of the parameter v used, the final costs tend to similar values, which correspond to the optimal.

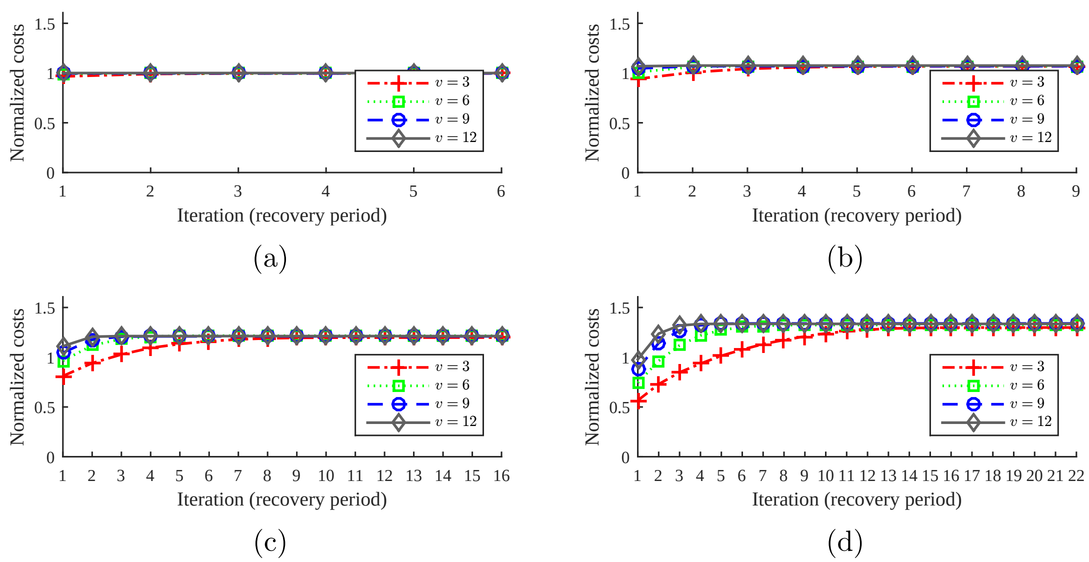


Figure 3.6 : Evolution of the flow costs, associated with earthquakes of magnitude $M_w = 6$ (a), 7 (b), 8 (c) and 9 (d). For each magnitude, the maximum number of repaired components by iteration $v = 3$, 6, 9 and 12 (González et al., 2016b).

The curves of reconstruction costs shown in Figure 3.7 are normalized by the cost associated with performing a full reconstruction (every node and arc) at once, since the cost of reconstruction of a fully destroyed system is greater or equal than the cost of reconstruction of any other destruction scenario. Then, the normalized reconstruction

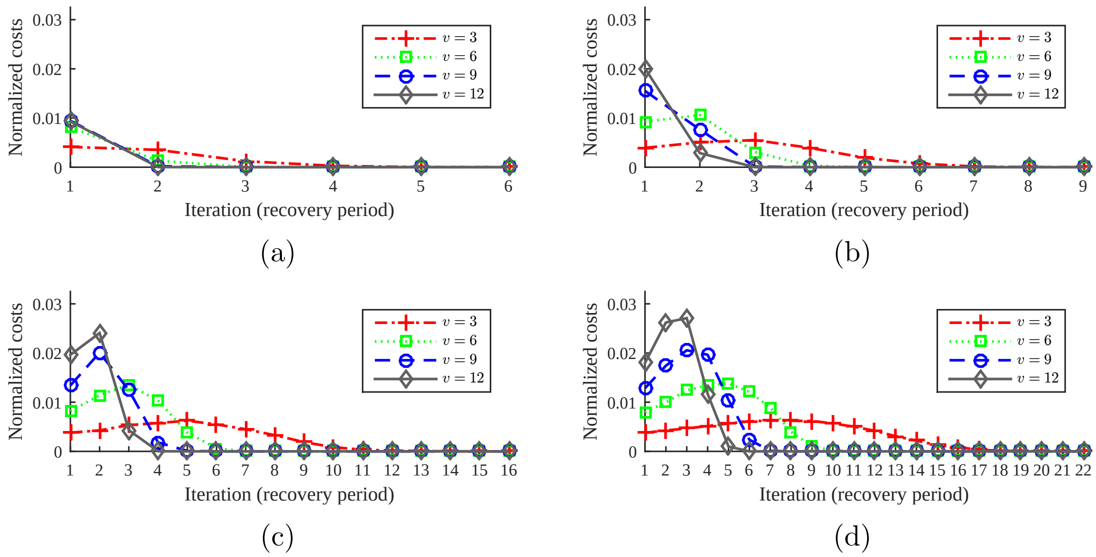


Figure 3.7 : Evolution of the reconstruction costs, associated with earthquakes of magnitude $M_w = 6$ (a), 7 (b), 8 (c) and 9 (d). For each magnitude, the maximum number of repaired components by iteration $v = 3$, 6, 9 and 12 (González et al., 2016b).

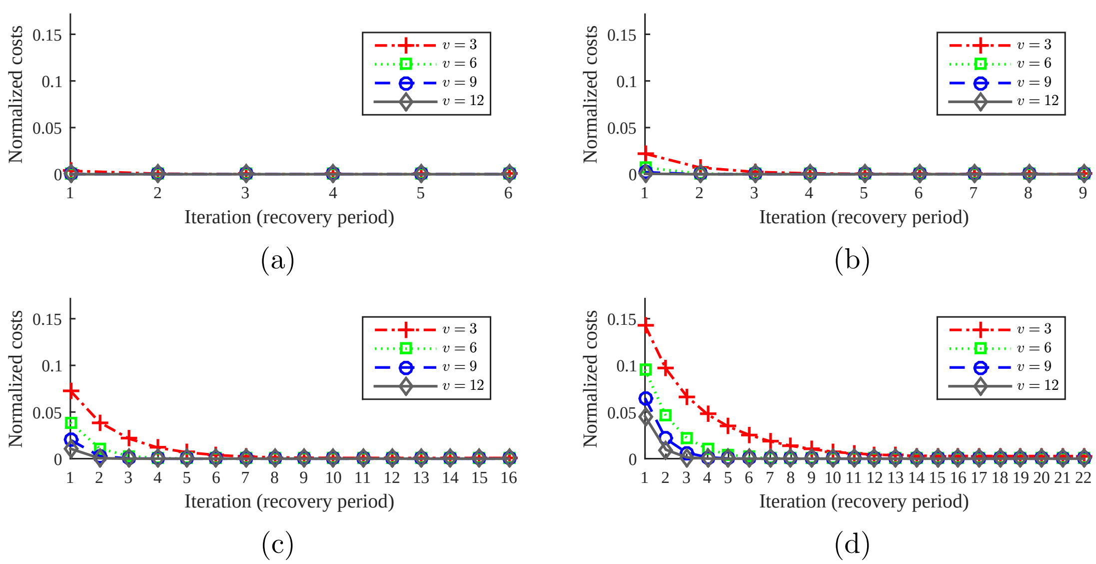


Figure 3.8 : Evolution of the total costs of recovery, associated with earthquakes of magnitude $M_w = 6$ (a), 7 (b), 8 (c) and 9 (d). For each magnitude, the maximum number of repaired components by iteration $v = 3$, 6, 9 and 12 (González et al., 2016b)

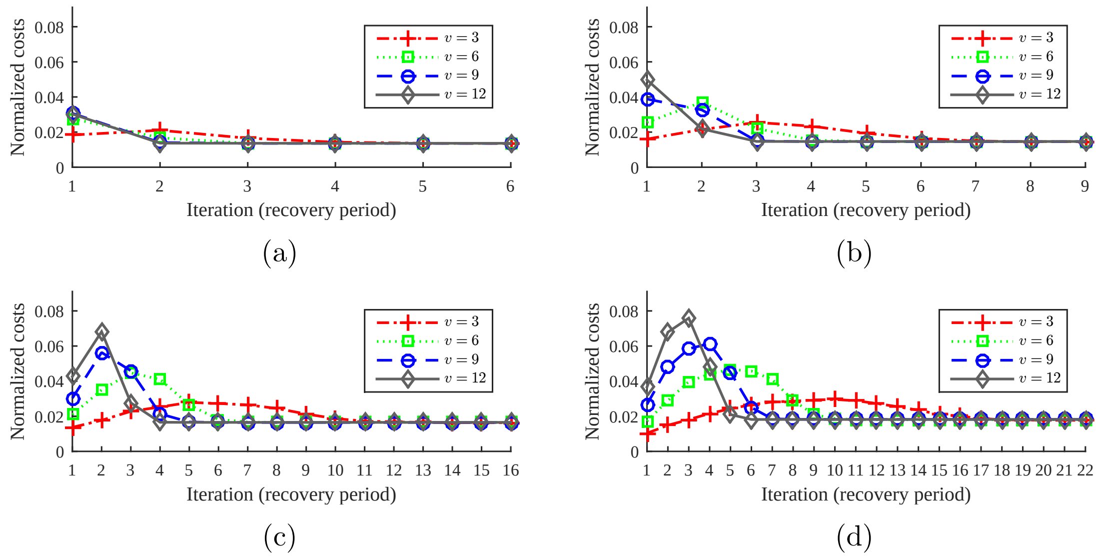


Figure 3.9 : Evolution of the total costs of recovery, without considering the unbalance costs, associated with earthquakes of magnitude $M_w = 6$ (a), 7 (b), 8 (c) and 9 (d). For each magnitude, the maximum number of repaired components by iteration $v = 3$, 6, 9 and 12 (González et al., 2016b)

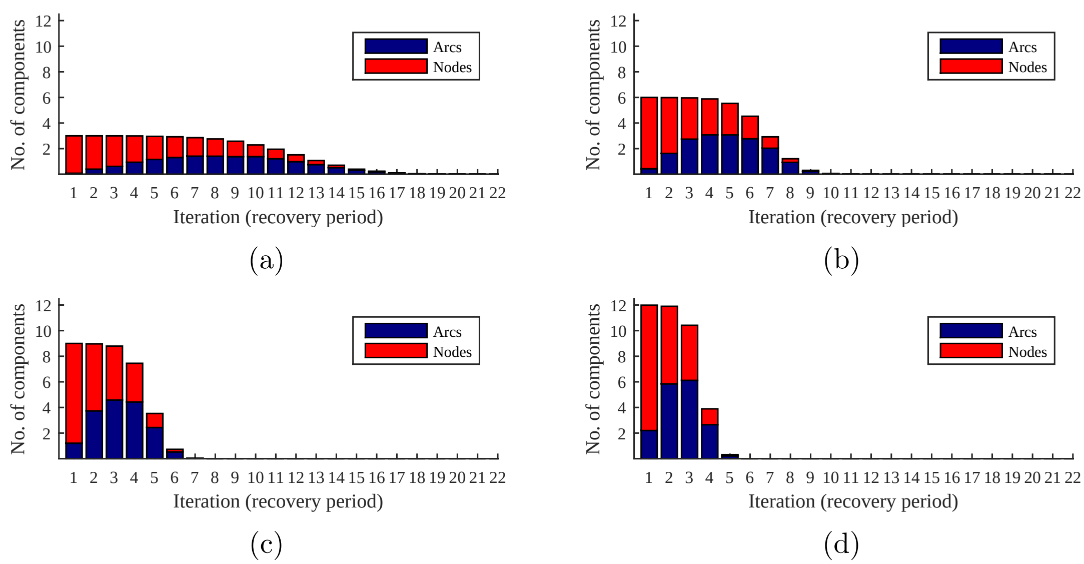


Figure 3.10 : Number of arcs (blue) and nodes (red) repaired on each recovery period, after an earthquake of magnitude $M_w = 9$, assuming maximum number of repaired components by iteration $v = 3$ (a), 6 (b), 9 (c) and 12 (d) (González et al., 2016b).

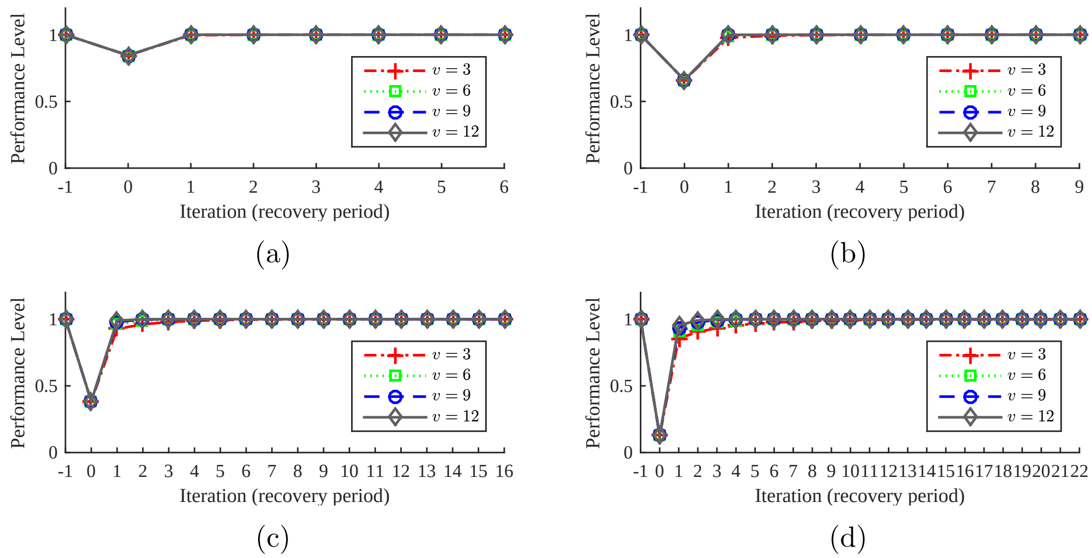


Figure 3.11 : Evolution of the total performance, associated with earthquakes of magnitude $M_w = 6$ (a), 7 (b), 8 (c) and 9 (d). For each magnitude, the maximum number of repaired components by iteration $v = 3$, 6, 9 and 12 (González et al., 2016b).

costs describe the fraction of the network that is being restored in each iteration. It can be seen that these costs decrease as the iterations increase; except for the initial iterations. This particular case corresponds to large earthquakes where there is a large number of destroyed components. This is explained by the fact that the iINDP assigns the cheapest high-importance recovery jobs at the beginning, and later some more expensive recovery jobs. Note that the initial values of the reconstruction costs in each magnitude studied do not differ much from each other, but as expected, for low-magnitude scenarios the complete reconstruction process is achieved much faster.

The total costs (flow, arcs and nodes reconstruction, unbalance and geographical preparation) associated with the recovery process are shown in Figure 3.8. These values are normalized by a reference value equivalent to the total costs associated with recovering a fully-destroyed network in just one iteration –obtained by solving the td-INDP assuming unlimited resources. Then, the results show that as the magnitude increases so does the initial total cost. Note that these costs correspond to the complete objective function (2.1a) of the td-INDP model; hence, it is to be expected that the model reduces as much cost as it can in every iteration. As desired, costs never increase, and the reduction rate is higher as the iteration is closer to the initial state, which is coherent to the fact that the iINDP model calculates the largest possible reduction in each iteration. Nevertheless, the unbalance costs used in the example are defined such that they are greater than any other cost (hence giving priority to ensure supply).

Figure 3.9 shows that independently of the magnitude of the earthquake or the amount of components repaired by iteration, the costs calculated with the iINDP have a steady convergence to a similar value, where the maximum difference between the calculated costs is only 0.3%. Such behavior is desirable, since it implies that independently from the initial conditions, the iINDP converges to a similar solution (in this case the optimal). This Figure summarizes the properties seen in the previous analysis, such as that the initial costs are in general much higher than the ones in the final stages (except in the high-magnitude cases where the initial disconnection does not allow full operation of the network), or that for largely destructed scenarios there is a fast initial growth in costs before all of them decrease to the final convergence value.

Figure 3.10 shows the average amount of nodes and arcs reconstructed in every iteration. As expected, the average amount of reconstructed components decreases as the iterations increase. Also, in early stages of the reconstruction process, focusing on the reconstruction of nodes is preferred to the reconstruction of arcs, which is an expected property considering that each node may have several arcs depending on it.

Finally, in order to study the resilience of the system, Figure 3.11 shows the evolution of the performance of the system. For this example, the performance level at a given time is defined as the unbalance between the total supply capacity and total demand, normalized by the total demand. For this Figure, recovery period '-1' indicates the period right before the earthquake, iteration '0' is related to the period when the earthquake occurs, and the remaining periods are related to the reconstruction process as before. As expected, the loss of performance is higher when the earthquake has a larger magnitude, being on average 15.5% for $M_w = 6$, 34% for $M_w = 7$, 61.5% for $M_w = 8$, and 87.3% for $M_w = 9$. It is possible to observe that the iINDP provides reconstruction strategies that promptly improve the performance right after the disaster, reaching in the worst case scenario ($M_w = 9$) a loss of performance less than 15% after just one iteration, 10% after two iterations, and less that 5% after four iterations. This exemplifies how the iINDP achieves its goal of quickly recovering the system to its original capabilities.

3.2 Analytical approach

Even though heuristic approaches such as the iINDP are useful to find good quality approximations to the optimal solution of the td-INDP, there are instances in which it may be important to solve the problem to guaranteed optimality. This Section focuses on analyzing the structure and properties of the td-INDP, in order to propose analytical approaches that can be used to solve it more efficiently, without losing guarantees of optimality. To this end, we start by performing a sensitivity analysis on the td-INDP, such that we can determine how its structure and solution times are affected by diverse parameters and network topologies.

3.2.1 Sensitivity analysis

The structure and computational complexity of the td-INDP model are related to diverse parameters, such as the probability of failure, the size of the system, the level of interdependency between networks, the availability of resources, and the number of periods in the planning horizon. In the previous Section we discussed how the time-complexity of the td-INDP increases exponentially with the number of periods $|\mathcal{T}|$. Keeping this in mind, let us focus on the case where $\mathcal{T} = \{1\}$, such that the td-INDP model focuses on determining the optimal recovery strategy for a single period ($t = 1$).

Considering this, we studied the behavior of the td-INDP model running time, as a function of the following analysis parameters:

- Failure probabilities: The more components are initially destroyed, the larger the number of constraints, since constraints (2.1g)-(2.1m) are defined for each destroyed component. Also, the more destroyed components, the more terms related to binary variables in the objective function.

- Nodes in the system of networks: The number of nodes studied determine the number of variables and equations considered, independently of the type of constraints.

- Links density: This term is defined as the ratio between the number of links in the system, and the maximum number of links for the studied network topology. The more links, the larger the number of flow variables.

- Interdependency density: This term is defined as the ratio between the components that depend on other components, and the total number of components in the system. The more dependent components, the larger the amount of constraints (2.1f).

- Interdependency strength: This term is related to constraints (2.1f), and refers to the average number of support components per dependent element. The more support components there are, the easier to enable the functionality of dependent components.

- Resource availability: Lower availability of resources implies less components that can be recovered. Then, it is expected that the less resources available the more difficult to find the optimal recovery strategy.

The study that follows assumes that there is a given system of interdependent networks, with fully defined sets and parameters according to the td-INDP formulation. Then, the exploration consists of varying each of the mentioned analysis parameters, such that it is possible to measure their impact on computational complexity and associated running times.

In order to obtain generalizable conclusions about the td-INDP time-complexity, this study does not focus on a single particular case, but studies different idealized topologies, some of which constitute the building blocks of realistic systems. In particular, this analysis focuses on the study of a multilayered system of two interdependent networks, with similar sizes and topologies. We selected three different topologies:

- Grid: Frequently found substrate in city-level transportation and utilities distribution networks.

- Wheel: Topology that is studied in communication networks (Mackenzie, 1966; Callander and Plott, 2005), where there is one driving node connected to the others. A wheel network with n nodes is composed by a single node connected to all nodes of a cycle of size $n-1$, resembling distribution within service areas.

- Erdős-Rényi: Related to graphs where all the links are randomly distributed. In this particular case, we generated these networks to have the same number of links of the grid, without ensuring connectivity.

Without loss of generality, the td-INDP input parameters were generated according to the following uniform distributions, considering that this study is intended to be general, with no prior information about the probabilistic behavior of the parameters: $b_{iklt} \sim U(-10, 10)$; $u_{ijklt} \sim U(0, 10)$; $c_{ijklt} \sim U(1, 10)$; $f_{ijkt} \sim U(1, 100)$; $q_{ikt} \sim U(1, 100)$; $g_s \sim U(1, 200)$; $M^{\pm}_{iklt} = 1000$; $h_{ijkrt} = 1$;and, $p_{ikrt} = 1$.

Such distributions describe a system with large variability, without bias or asymmetry. For each topology and parameter configuration, the number of Monte-Carlo simulations was fixed to 1000. The upper and lower bounds are chosen such that in average the demands and capacities have similar magnitudes. Also, such that the cost of preparing reconstruction jobs for multiple co-located components is on average greater than the cost of repairing a single component. Note that the costs are never zero or less, implying that all flows and repairing processes would increase the objective function. This analysis was based on a multilayered system (Boccaletti et al., 2014), where each component from one network has exactly one correspondent component in the other network. Thus, it was assumed that repairing a component implied repairing a unique space $s \in \mathcal{S}$ specific to each component.

For all topologies, the system analyzed had the following default analysis parameters: Failure probabilities = 0.5; order of the system = 32 nodes; links density = 0.5; interdependency density = 0.5; interdependency strength = 0.5; and, resource availability = 6 units.

Figure 3.12a shows the running times for the studied topologies as a function of the failure probabilities. As expected, the mean running times increased with the probability of failure. Nevertheless, such an increment appears to be linear. Similarly, the variability of the running times is also higher for larger failure probabilities. This is expected, as the probability of failure is directly linked to the number of destroyed components.

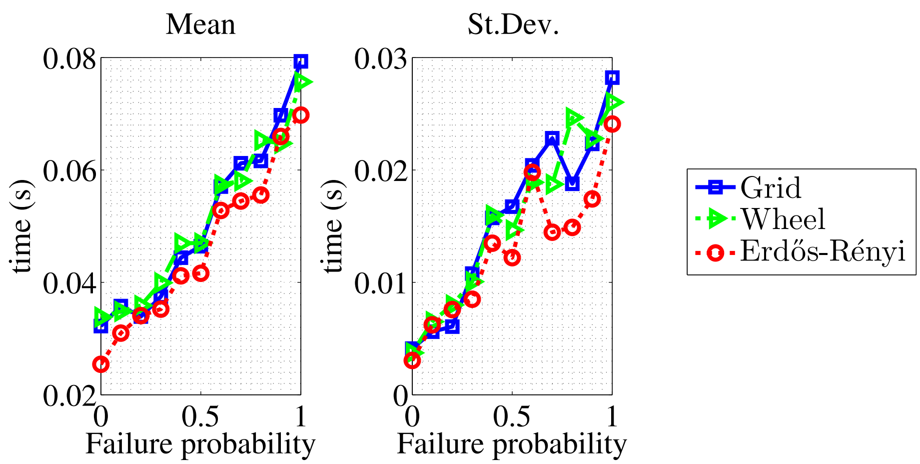
Similarly, Figure 3.12b shows the mean running times for diverse system sizes, where it can be seen that the times (and their variability) seem to also increase linearly with size. Figure 3.12c shows that, contrary to what one could expect, the link density of the undamaged system appears to have no major impact on the running times. This is explained by the fact that the constraints that include binary variables (which are the ones that drive most of the complexity) are not related to the number of links, but to the number of destroyed links and nodes.

Figure 3.13a shows that the larger the interdependency density, the larger the running times. This was also expected, as the interdependency density determined the number of constraints related to the physical interdependence. Similarly, Figure 3.13b shows that the running times tend to increase with the interdependency strength. Nevertheless, observe that this trend is almost non-existent for the grid topology. Such a behavior may be related to how redundant and uniform the system's structure is, since the grid topology is in general highly redundant, whereas the other topologies may get disconnected easier due to failing components.

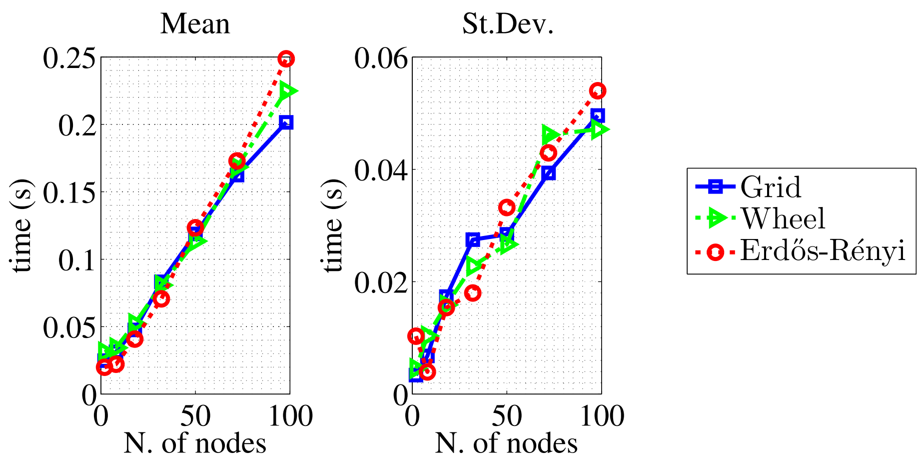
Finally, Figure 3.13c shows that the running time is highly dependent on the resource availability. Nevertheless, as opposed to the other scenarios, the relation between the running times and the analysis parameter is not monotonic. In this case, the complexity increases vastly as the resources increase, but only for resource levels up to a value of approximately six. From that value on, the running times slightly decrease with the resource availability. This is a most interesting behavior, related to the ratio between the available resources v_r and the resource utilization (h_{ijkrt} and p_{ikrt}). Given that resource utilization was assumed to be one in this experiment, v_r could be seen as the number of components that could be fixed. Note that to fix one link that is dependent on another, one needs to ensure functionality of at least the parent link with its starting and ending nodes, and the dependent link with its starting and ending nodes. This corresponds to six components, which is precisely the found threshold.

In general, it was possible to observe that the time-complexity of the td-INDP model increased with the size of the networks, the density and strength of interdependencies, the resource availability (when resources are too limited), and the probability of failure. Nevertheless, it was possible to observe that the time-complexity did not seem to dependent greatly on the studied topology, except when studying the interdependency strength, where a less reliable topology implied a larger negative impact on the running times.

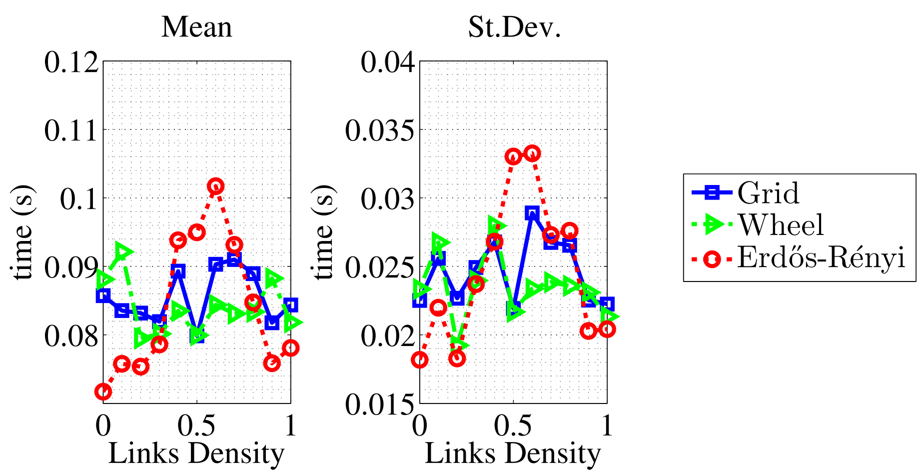
To give a sense of how the td-INDP structure depends on the underlying topology studied, Figure 3.14 shows the matrix representations of the td-INDP constraints for a each of the three topologies studied (assuming the standard configuration). Observe that these matrices have almost identical structures, implying that the td-INDP model may not depend strongly on the topology of the studied networks. Considering this, the next Section provides a more detailed description of the structure of the td-INDP, and presents possible analytical decomposition approaches that could be used to exploit such a structure.



(a) Failure probability

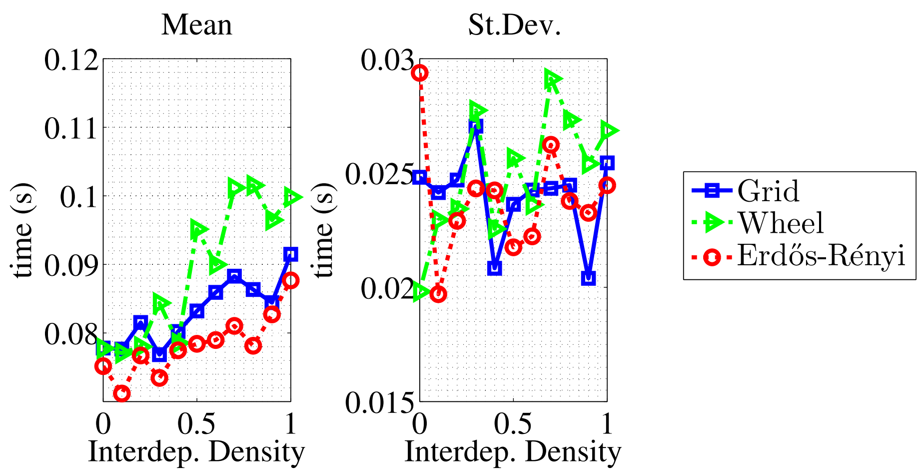


(b) Number of nodes in the system

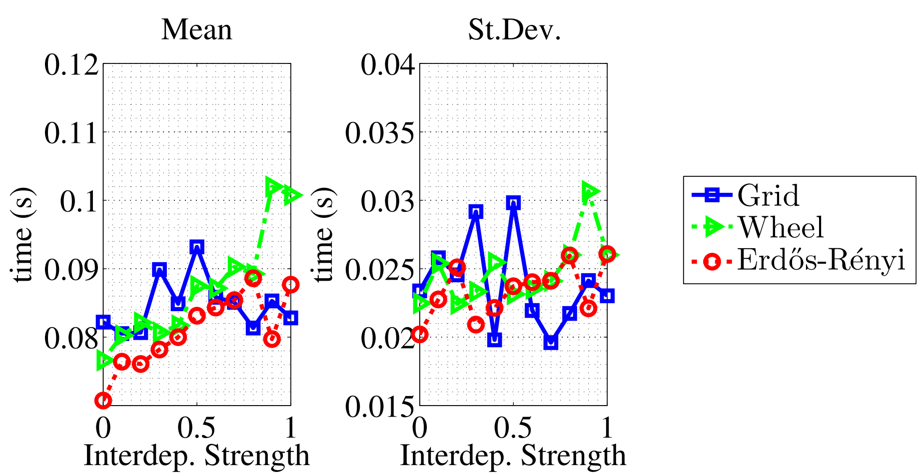


(c) Average link density in the system

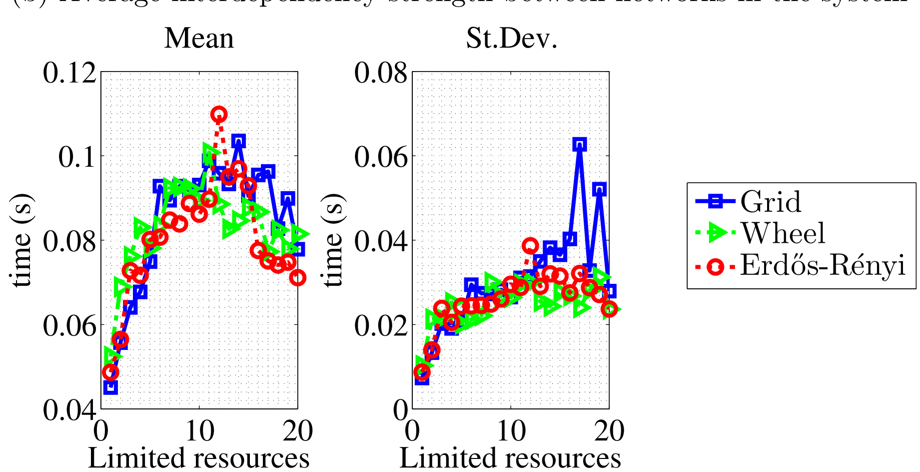
Figure 3.12 : td-INDP time-complexity sensitivity to different topology-related parameters (González et al., 2015).



(a) Average interdependency density between networks in the system



(b) Average interdependency strength between networks in the system



(c) Resource availability for recovery

Figure 3.13 : td-INDP time-complexity sensitivity to different interdependence-related parameters (González et al., 2015)

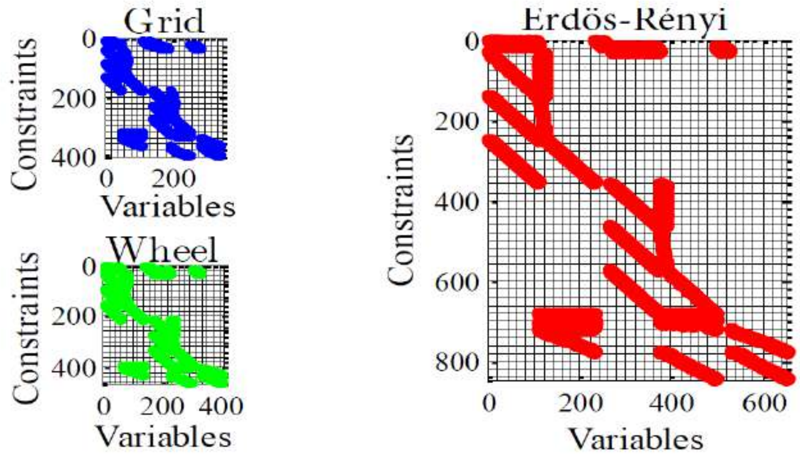


Figure 3.14 : td-INDP coefficient matrices for the three studied topologies, with standard configuration (González et al., 2015).

3.2.2 Analytical decomposition strategies

In general, Mixed Integer Linear Programs (MILP) belong to the NP-hard computational complexity class. However, depending on the specific structure of the MILP to be solved, there may be analytical approaches that permit reducing their expected solution times. In particular, techniques such as Dantzig-Wolfe (Dantzig and Wolfe, 1960) and Benders (Benders, 1962) decompositions have been successful in reducing the average solving times of diverse network flow problems, such as the Minimum Cost Flow Problem (MCFP) (Tomlin, 1966; Vaidyanathan, 2010; Hamacher et al., 2007), the classical Network Design Problem (NDP) (Poss, 2011; Dionne and Florian, 1979), and subsequent extensions of them. Given their relevance for our td-INDP developments, a detailed literature on these decomposition techniques is described below.

Dantzig-Wolfe decomposition

This decomposition was introduced by Dantzig and Wolfe (1960), as a decomposition technique to specifically target linear programs. Dantzig and Wolfe (1960) proposed a decomposition that reformulates a linear program into two subproblems, a master problem and a pricing problem, that are iteratively solved and coordinated. Dantzig and Wolfe (1960) showed that this decomposition always converges in a finite number of iterations, and that its final result corresponds to the optimal solution of the original linear problem. Later, Wollmer (1969) showed that Multicommodity Minimum Cost Flow (MMCF) problems could be solved using the aforementioned decomposition. In particular, Wollmer (1969) showed that MMCF formulations could be separated in a block-diagonal structure, except for a set of constraints that coupled all these blocks. Taking advantage of this property, Wollmer (1969) constructs a Dantzig-Wolfe solution scheme, that coordinates the iterative solution of two separate sub-problems, one associated with the coupling constraints and the other with the fully block-diagonal structure. Since then, a plethora of studies have shown that Dantzig-Wolfe decomposition schemes are usually successful for linear problems that, like the MMCF, are composed of block-diagonal coefficient structures and a set of constraints that couples them together (Assad, 1978; Kennington, 1978; Jones et al., 1993). In recent years, diverse studies have shown further applications of the Dantzig-Wolfe decomposition on diverse network flows and network design problems. For example, Lin et al. (2011) developed a heuristic based on Dantzig-Wolfe decomposition to solve a bi-level Network Design Problem (NDP). The master problem corresponds to a linear budget allocation problem, whereas the pricing problem relates to flow assignment. Also, Frangioni and Gendron (2013) introduced the stabilized structure Dantzig-Wolfe decomposition, and showed its application to solve the multicommodity capacitated network design. Ciappina et al. (2012) also showed how to apply the Dantzig-Wolfe decomposition to solve multicommodity flow problems, but considering fuzzy costs in the formulation. For general purposes, Bergner et al. (2014) showed how to automatically reformulate mixed integer programs using Dantzig-Wolfe decompositions, emphasizing that the quality of the reformulation highly depends on the structure of the original problem, where network flow problems (such as INDP-related problems) tend to be well suited.

Benders decomposition

This decomposition was introduced by Benders (1962), where he shows that mixed-variables programming problems could be assessed by iteratively solving two coordinated sub-problems. In particular, he showed that this decomposition could be used to solve a Mixed Integer Linear Program (MILP), by constructing a sub-problem associated with the integer variables, and the other sub-problem with the continuous variables. Later, Magnanti et al. (1986) introduced conceptually the applicability of Benders decomposition for uncapacitated network design, based on the generation of Benders cuts for the NDP. Since then, multiple studies have shown the applicability of Benders decomposition in network flow problems. For example, Sridhar and Park (2000) showed how to use Benders cuts and polyhedral cuts for capacitated network design in a branch-and-bound fashion (Benders-and-cut). They showed that the proposed algorithm has a small integrality gap for low demands, but for large demands in the system the gap increases, and the algorithm becomes slow. Binato et al. (2001) showed an application of Benders cuts for the NDP in power transmission problems. In order to improve the efficiency of the proposed approach, they also include Gomory cuts. Costa (2005) presented a survey on Benders decomposition for fixed-charge NDP, concluding that, in this context, Benders decomposition may outperform other traditional techniques, such as Lagrangian relaxation and Branch-and-Bound. Costa et al. (2007) studied the use of Benders, metric, and cutset inequalities to solve the multicommodity NDP, where they observed that the use of strengthened Benders inequalities highly accelerated to solution times. Fortz and Poss (2009) presented an extension of Benders decomposition for a multi-layered NDP, showing the applicability of Benders inequalities in a system of interconnected networks.

Based on the reviewed literature, we showed that the Dantzig-Wolfe and Benders decompositions are usually well suited for network flow problems such as the MCFP and the NDP. In the following Section we show that the proposed td-INDP can be seen as an extension or a generalization of these well known problems, thus making it susceptible to the aforesaid decompositions.

The structure of the td-INDP

In order to show that the proposed td-INDP is a generalization of the MCFP and the NDP, let us study the following special cases of the td-IDNP. First, assume the simple case where there is only one network. Now, consider a context where there is no set of limiting resources, and the geographical preparation costs are negligible. Then, the td-INDP would be solving a MCFP for period $t = 0$. Also, under this context, variable Δz_s would not be relevant, and constraints (2.1k), (2.1l), (2.1m) and (2.1v) would not exist. On one hand, if the studied network is fully functional (i.e., there are no destroyed components), then $\Delta y_{ijkt} = 0$ and $\Delta w_{ikt} = 0$, for all nodes and arcs, which in turn implies that $\Delta z_{st} = 0, \forall s \in \mathcal{S}$. Then, only constraints (2.1b)-(2.1e) would take place and the problem would not be MIP anymore, as all the integer variables are known beforehand. Then, under this configuration, the td-INDP would be solving multiple instances of a MCFP (one per period), which is solvable in polynomial time (Orlin, 1997). On the other hand, if we assume a single partially destroyed network, only constraints (2.1b)-(2.1e) would take place again. Nevertheless, Δy_{ijkt} and Δw_{ikt} would be unknown. Then, under this configuration, the td-INDP model would be solving a classical NDP for each time period (with additional time-coupling constraints). Then, if solving for one recovery period, then the td-INDP could be solved by first solving a MCFP for $t = 0$, and then a NDP for $t = 1$, making the td-INDP NP-complete in this configuration (Johnson et al., 1978).

Now, assume that there are multiple interdependent networks being studied. Then, the td-INDP model would be solving coupled NDPs for each network (and period). The more interdependent the networks are, the more constraints (2.1f) will take place, thus increasing the complexity of the overall model. Likewise, including a set of limiting resources would also increase the complexity, as constraints (2.1k) depend on the binary variables Δy_{ijklt} and Δw_{iklt}. Finally, including geographical preparation costs into the analysis would further increase the complexity of the MIP model, as it depends on the additional binary variables Δz_s.

From the previous line of reasoning, we can conclude that the full td-INDP model, which considers interdependence between networks and also geographical preparation costs related to binary variables Δz_s, would indeed be in general NP-hard. However, we can also conclude that the td-INDP can be described as a set of multiple coupled MCFP and NDP formulations. Thus, the described coupled techniques could be applied to decompose the td-INDP into multiple MCFP and NDP sub-problems, which in turn may also be further decomposed. Furthermore, if we assume that the studied system allows partial functionality states (w_{iklt} and y_{ijklt}), then only the recovery variables (Δw_{iklt}, Δy_{ijklt}, and Δz_{st}) would be binary. Then, a decomposition approach could be applied to separate the td-INDP into recovery- and operation-oriented sub-problems.

Figure 3.15 illustrates the coefficient matrix associated with the td-INDP, showcasing the special structure described above. From this Figure, it is easy to see that if the recovery variables are the only binary variables, then we could use Benders to separate the td-INDP into a sub-problem associated with the recovery of the system, while the other sub-problem would focus on the operation of the system (determining the flows of commodities, the functionality of each element, and the over/under supply). The operation-oriented sub-problem could be then decomposed using Dantzig-Wolfe decomposition, where one sub-problem would be associated with the operation of the system at each individual period, and the other sub-problem with the time-coupling constraints (the constraints that guarantee that any recovery done at time t is affecting the operation at time $t+1$).

Each of these sub-sub-problems, represented in 3.16, would then be associated with the operation of the system at each time. Note that this could be further decomposed considering the different networks being modeled, and the interdependencies between them.

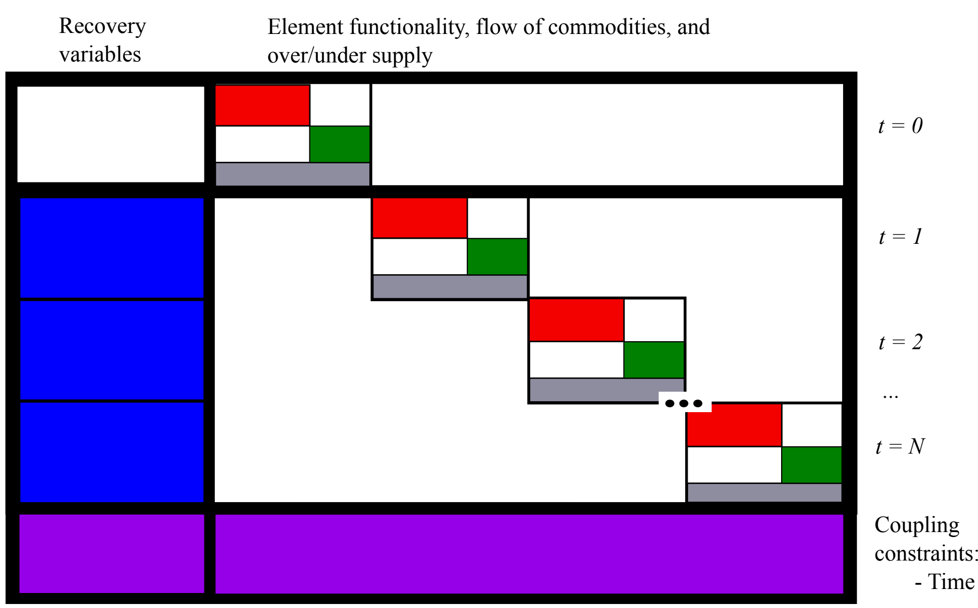


Figure 3.15 : Graphical representation of the coefficient matrix associated with the td-INDP

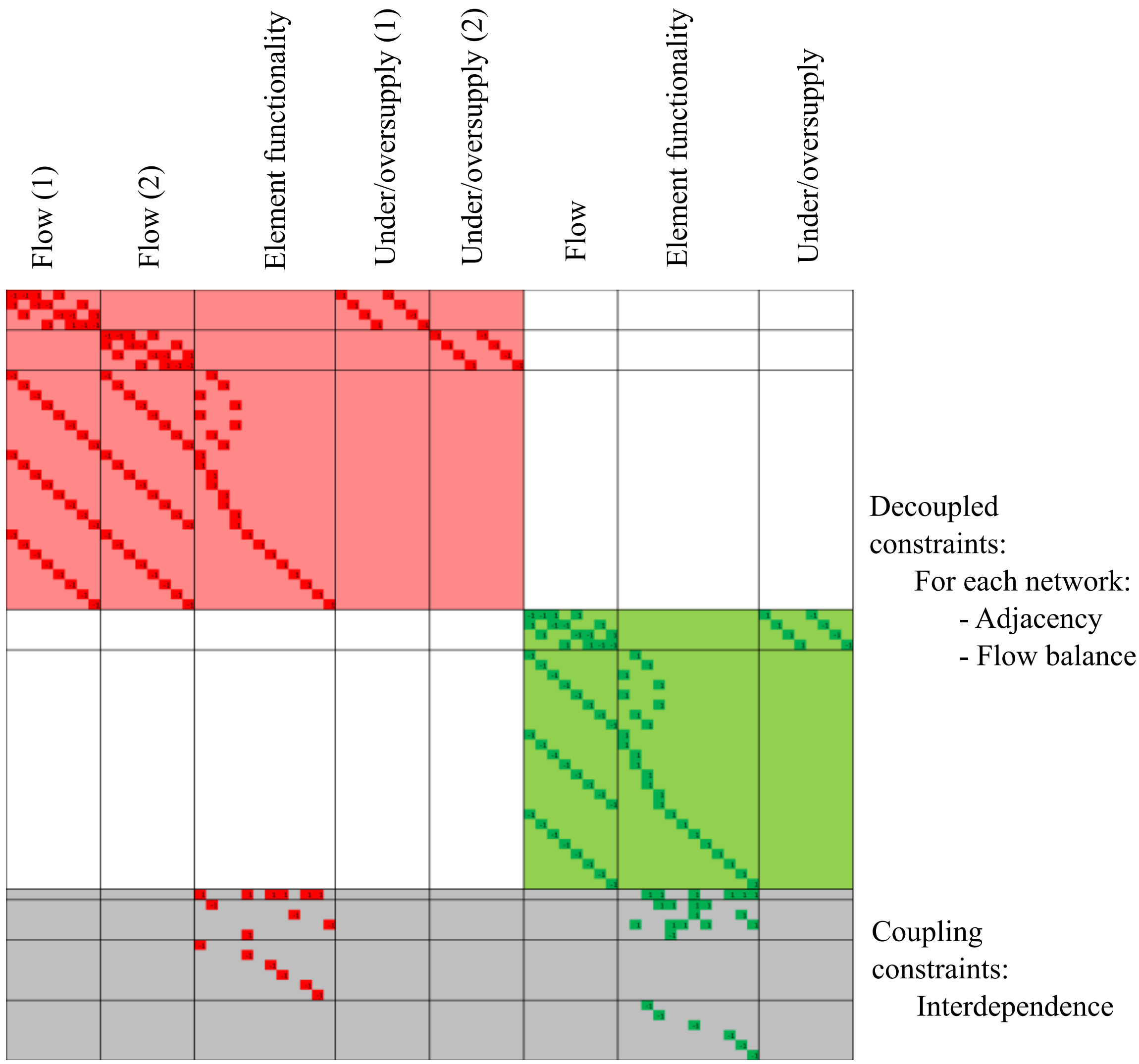


Figure 3.16 : Decomposition of the coefficients associated with element functionality, flow of commodities, and over/under supply, for each recovery period (corresponding to a block off of Figure 3.15).

3.3 Conclusions

In this Chapter we proposed diverse approaches that can be implemented to accelerate the td-INDP. We showed that the td-INDP solving times increased exponentially with the number of recovery periods considered in the planning horizon. Considering this, we proposed a heuristic approach, denominated the iINDP, which decomposed the planning horizon into multiple smaller horizons and iteratively solved a reduced td-INDP for each of them. By solving for separate time horizons at each iteration, the iINDP can include updates on the element capacities, flow and recovery costs, resources, and even the damage states of each element. This enables, among other things, modeling problems that consider multiple damaging events, such as earthquakes with subsequent aftershocks. Additionally, we showed that with this heuristic approach the solving times only increased linearly as a function of the periods in the planning horizon. This allows performing simulation-based studies in a time-efficient manner, as these require solving the several multiple damage and recovery instances. To showcase the capabilities of the iINDP, we performed a Monte-Carlo-based analysis on the system of power, gas, and water networks in Shelby County, TN., to study its expected resilience, associated with an earthquake hazard. We showed that this approach may also be used to study the expected recovery and operation costs, constituting an insightful tool for decision makers and other stakeholders, regarding post-event endeavors. However, note that the proposed simulation approach could also be used for pre-event analysis, to support mitigation- and preparedness-oriented studies. Such an analysis is presented in Section 4.1. Also, note that the proposed iINDP was designed such that it could be easily implemented to enhance any interdependent network recovery formulations beyond the td-INDP, such as the formulations proposed by Lee II et al. (2007) and Cavdaroglu et al. (2011).

We also performed a comprehensive sensitivity analysis on the td-INDP, which showed that its time complexity is highly dependent on the size of the analyzed system of networks, its level of damage, and availability of resources, but not as much on its link density, or its interdependency density and strength. Additionally, we showed that, independently of the studied networks' topologies, the proposed td-INDP depicts a special structure, suitable for decomposition techniques, such as Dantzig-Wolfe and Benders decompositions. This special structure, along with its respective proposed decompositions, are further studied in Section 4.2, where we extend the proposed td-INDP to include uncertainty in its parameters.

Chapter 4

Modeling Uncertainty

As we have discussed in previous sections, in order to quantify and optimize the resilience of any given system, it is not only important to take into account the constraints associated with its operation, but also to consider the different sources of uncertainty that may impact the system's operation and recovery*. Among the several diverse sources of uncertainty that affect the resilience of a system, there are two that are prominent, particularly when studying realistic networked infrastructure systems:

- Uncertainty related to the failure modes of the system, their causes, and furthermore, their implications.
- Uncertainty related to the physical and logical properties of the system, and possible errors quantifying them.

Note that the MIP formulation that was defined at the core of the proposed research, the td-INDP (Section 2.2.2), is fully deterministic. This means that the td-INDP formulation assumes that all physical and logical properties of the studied system are fixed and known, as well as the initial damage state. Then, in order to include the mentioned sources of uncertainty into the analysis, it is necessary to extend the capabilities of the current formulation. To this end, there are two major approaches that could be used. On one hand, it is possible to rely on sampling-based

*This Chapter is based on the ideas and contents presented in González et al. (2014b), González et al. (2014a), González et al. (2017b), and the working paper González et al. (2017c). The definitive versions of these papers are available at ascelibrary.org and www.taylorandfrancis.com methods, such as Monte-Carlo simulation (Rocco S and Zio, 2005; Cancela et al., 2013), along with associated techniques such as importance sampling (Hurtado, 2007) and latin-hypercube (Olsson et al., 2003), among others. On the other hand, it would be possible to use optimization techniques with embedded uncertainty, which include robust (Sim and Bertsimas., 2003; Beyer and Sendhoff, 2007; Gao and Zhang, 2009) and stochastic optimization (Waller and Ziliaskopoulos, 2001; Hong et al., 2015).

In this Chapter, we show how these approaches can be used to extend the td-INDP model capabilities, in order to consider the two main sources of uncertainty discussed. First, Section 4.1 presents an approach to study the recovery and resilience of interdependent systems, using the td-INDP while considering uncertainty associated with failure of components. This approach builds upon the sampling-based study introduced in Section 3.1. Later on, Section 4.2 presents a new approach that enables extending the td-INDP model using stochastic optimization, to embed the uncertainty associated with characteristics of the studied system of networks, such as unknown demands and resources. In particular, we present a new stochastic MIP formulation to solve the INDP, denominated the stochastic INDP (sINDP) model, along with an illustrative example associated with the power, gas, and water networks in Shelby County, TN.

4.1 Uncertainty regarding the failure of components

Uncertainty regarding failure of components of a given system —such infrastructure networks— may come from the possible occurrence of a natural destructive event, aging and deterioration of the system components, or even due to malicious attacks and human errors in the operation. Related to the occurrence of natural events such as hurricanes, floodings, or earthquakes, it is common to use fragility models that are based on both historical data and analytical studies on the physical properties and response of the system components (Adachi and Ellingwood, 2009, 2010). Following this concept, the U.S. Department of Homeland Security develops manuals and relevant software to calculate the fragility of diverse types of structures and facilities (FEMA, 2013). Regarding deterioration modeling, there are several techniques to study and depict aging and degradation processes (Sanchez-Silva et al., 2011), including Bayesian updating (Coluccia, 2011; Boudali and Dugan, 2005; Morales-Nápoles et al., 2013) and Markov Decision Processes, which also study intervention strategies, among others (Alagoz et al., 2010). In this study, we focus primarily on analyzing the failure caused by destructive events, considering that these are external to the system and their occurrence cannot be directly controlled.

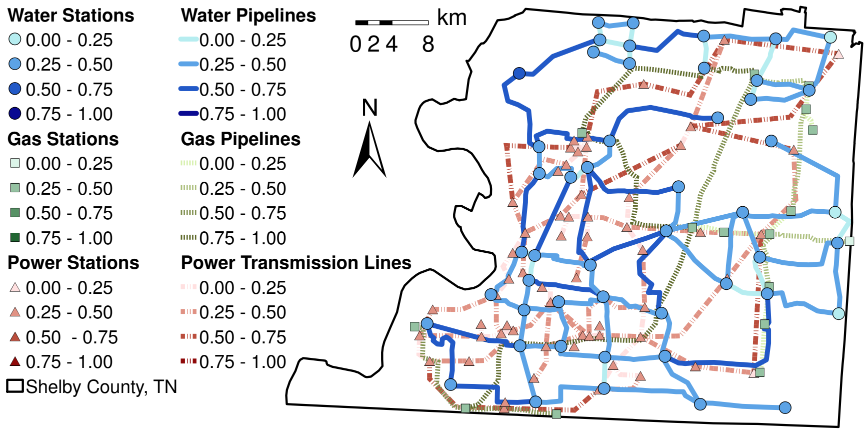
To study the uncertainty associated with the failure modes of the system, we can directly apply sampling-based techniques in conjunction with the proposed td-INDP model, since the uncertainty would be taken into account by the sampling process, and not by the mathematical optimization model, which was designed to be a post-event optimization tool. For example, the illustrative example presented in Section 3.1 (González et al., 2016b) estimated the expected performance and recovery costs of the water, gas, and power networks in Shelby County, TN, subject to an earthquake hazard due to its proximity to the New Madrid Seismic Zone (NMSZ). In this Monte-Carlo approach, we simulated multiple damage realizations consistent with earthquake magnitudes $M_w \in \{6, 7, 8, 9\}$, and determined their subsequent recovery strategies using the iINDP. As previously mentioned, from the variety of methods available to calculate the failure probabilities of the network's components due to earthquakes, we adopted the fragility models presented by Adachi and Ellingwood (2009) and Adachi and Ellingwood (2010), given that they studied similar infrastructure systems and hazards to the ones proposed in this example. In this case, failure probabilities were calculated based on the location and magnitude of the earthquake, and the Peak Ground Velocity (PGV) and Peak Ground Acceleration (PGA) on the affected areas, among other characteristics.

This study shows that the td-INDP, despite being designed as a post-event tool, can be used to perform pre-event analysis, to support budget planning and help evaluating the possible impacts of a given disaster. Expanding on the idea of performing td-INDP-based pre-event analyses, we perform a computational study on Shelby County with the objective of providing component-wise information that would help supporting mitigation and emergency planning (González et al., 2014b). This study is centered on measuring diverse component-wise performance metrics, to inform stakeholders about their vulnerability and relative importance for resilience.

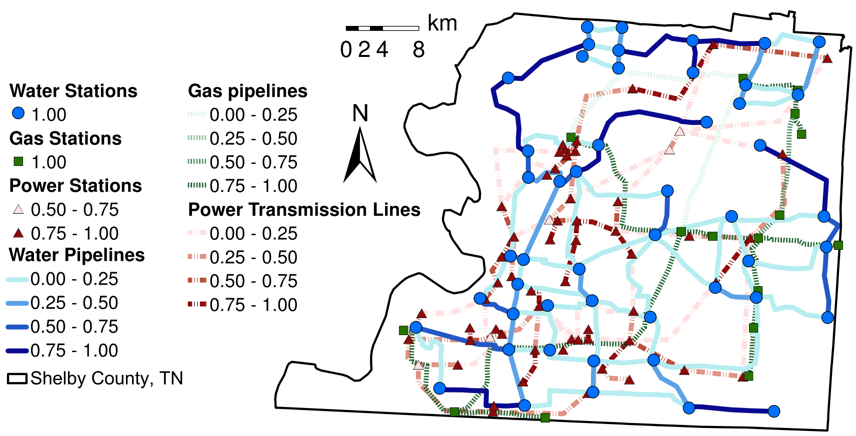
Resilience metrics

For any given networked system, there are three component-wise metrics of high relevance that are often studied, which relate to the resilience of the system: the failure likelihood of each component; the recovery likelihood of each component; and, the recovery time of each component. Following the simulation-based study introduced in Section 3.1, we estimated each of these resilience metrics for each component. As expected, we observed that each of the three proposed metrics describe different dynamics from the system of networks, hence the importance of studying all of them. In order to showcase the different dynamics observed, Figure 4.1 depicts the three aforementioned resilience metrics, related to an earthquake of magnitude $M_w = 9$.

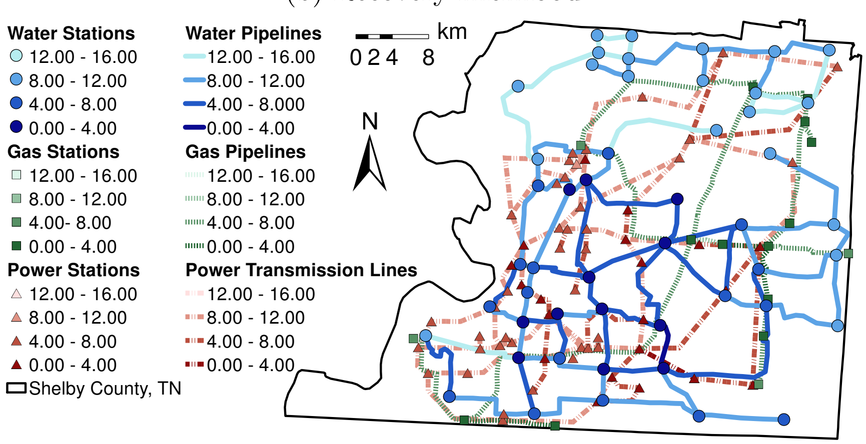
Figure 4.1a shows the likelihood in which each component would fail subject to the earthquake. As expected, it can be seen that this metric is related to the physical properties of each component (such as its size), since the longer the linear components are, the more prone to damage they are. However, it can also be observed that there



(a) Failure likelihood



(b) Recovery likelihood



(c) Recovery times (periods)

Figure 4.1 : Resilience metrics for critical infrastructure networks in Shelby County, TN. (González et al., 2014b)

is a specially high correlation between the failure likelihood and the location of the earthquake (located at the Northwest area of the county). Thus, it could be said that this metric is mostly guided by the hazard.

Figure 4.1b shows the likelihood in which the recovery of a given component is part of the optimal recovery strategy. In this case, it is possible to see that mostly those components that have little connectivity redundancy (connecting isolated nodes) are the most critical ones. This is expected, as clearly redundancy provides more options to supply a given area with the demanded commodities. Thus, it is said that this resilience metric is mostly related to the topology of the system.

Finally, Figure 4.1c shows the average time in which each component is recovered (if initially damaged). It can be seen that the nodes located at the center of the depicted networks are the ones with higher priority for fast recovery. Note that this is where most of the demands in the system are generated, since this is where the city of Memphis is located. Thus, it is clear that this metric is mostly driven by the demand configuration of the system.

Whenever studying realistic systems, it is important to ensure that the proposed resilience metrics return consistent results, independently of the magnitude of the simulated earthquake, since forecasting the magnitude of an earthquake accurately is usually not feasible. To evaluate this consistency, we can assume that each resilience metric provides a different ranking for the components in the system, such that we can evaluate if the importance sorting proposed by each ranking is similar, independently of the disaster level. To evaluate if the proposed resilience metrics are consistent for different earthquake magnitudes, we compared the estimated metrics created based on disaster realizations consistent with the four different proposed magnitudes ($M_w \in \{6, 7, 8, 9\}$). Figure 4.2 shows the correlation profile for the node failure-likelihood metric evaluated for each magnitude. In particular, Figure 4.2 shows the

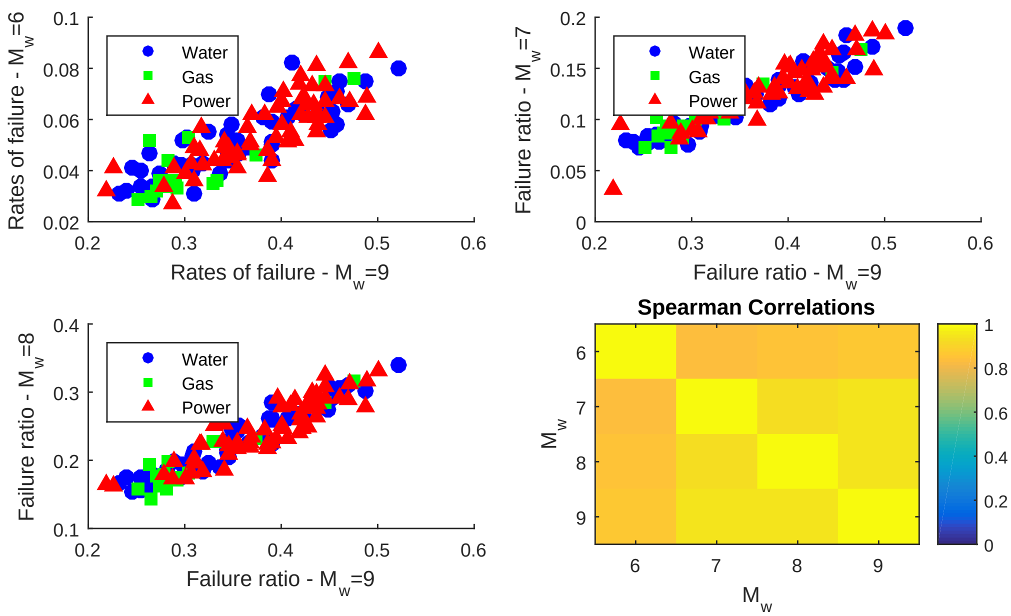


Figure 4.2 : Correlations between node failure ratios associated with different earthquake magnitudes

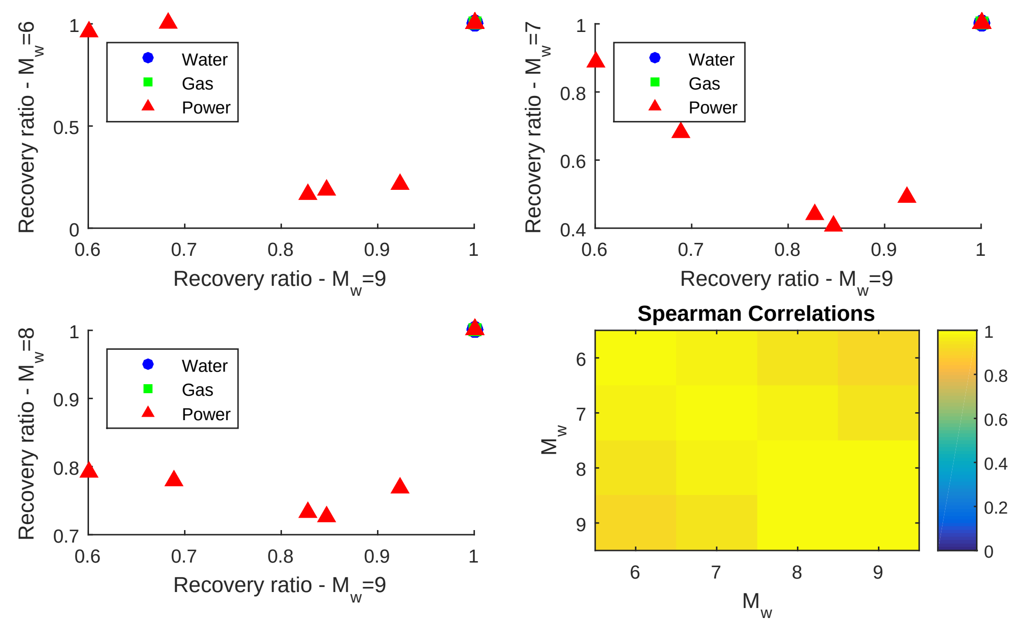


Figure 4.3 : Correlations between node recovery ratios associated with different earthquake magnitudes

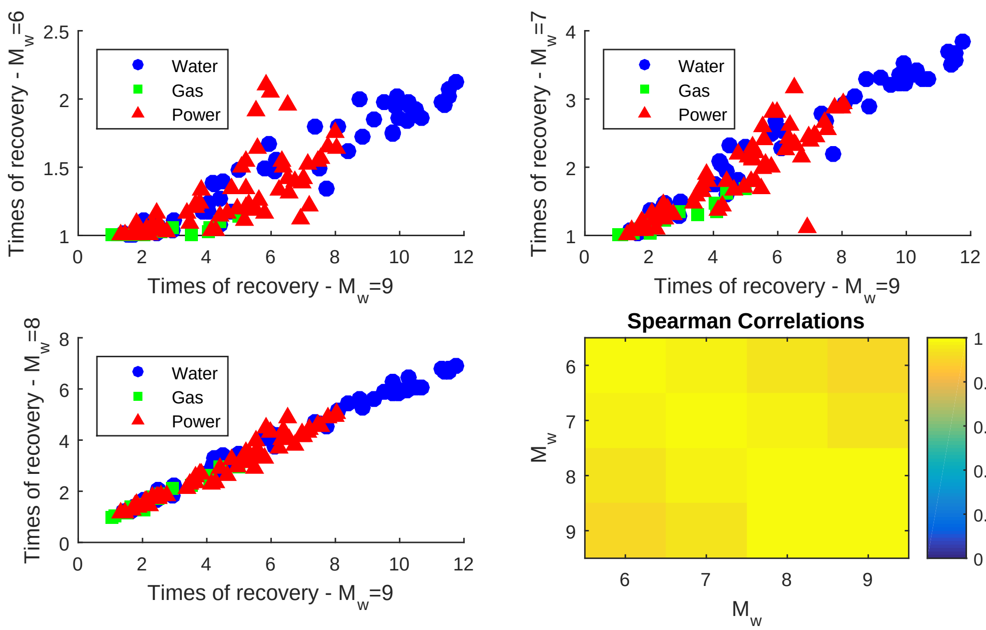


Figure 4.4 : Correlations between node recovery periods associated with different earthquake magnitudes

scatter diagrams for three different pair-wise comparisons ($M_w = 6$ vs $M_w = 9$, $M_w = 7$ vs $M_w = 9$, and $M_w = 8$ vs $M_w = 9$), as well as the Spearman rank-order correlation between all possible combinations. From Figure 4.2, it can be seen that the proposed metrics are highly correlated, both in value and order. Thus, as desired, the failure-likelihood metrics show they are consistent independently of the disaster level simulated.

Likewise, Figures 4.3 and 4.4 show the correlation profiles for the recovery-likelihood and recovery-time metrics, respectively, also associated with the nodes. As desired, these Figures show that metrics' values and their associated rank-orders are highly correlated, showing that these are consistent as well. Similar high-correlation profiles were also observed when studying the arcs, which indicates that their associated resilience metrics were also consistent, independently of the type of element studied.

Considering that the proposed resilience metrics return consistent ranking values for both types of components, we may infer that such metrics describe intrinsic properties of the studied system of interdependent network, as well as the hazard effect on them. Note that Figure 4.3 shows that the recovery likelihood is high for the majority of nodes (where most of them reach values of 1, indicating that they are always recovered). This indicates that recovering nodes is a priority to maintain adequate operation of the system, in agreement with the findings made in Section 3.1.

Since we just showed that all the three proposed resilience metrics return consistent rank-orders, we can conclude that these metrics determine importance rankings associated with three relevant 'dimensions' of the studied system: the studied hazard; the topology of the system of networks; and, the demand of commodities in the system. Thus, failure-likelihood ranking could be used to support decision makers and stakeholders in determining which elements are more prone to failure, such that they can retrofit them. Likewise, the recovery-likelihood ranking could be used to indicate which areas of the system need to be redesigned or upgraded, to increase their capacities and redundancies. Finally, the recovery-time ranking could be used to estimate ideal locations for emergency response and resource supply centers, based on the critical areas where the system needs to be recovered the fastest, such that most of the demand (thus, performance) can be covered adequately after a disaster event.

4.2 Uncertainty regarding the system properties

In recent years, researchers have developed diverse mathematical methodologies to understand, quantify, and optimize the the recovery of systems of interdependent networks –such as Lee II et al. (2007), Cavdaroglu et al. (2011), and the td-INDP model presented in Section 2.2.2 (González et al., 2016b)–, acknowledging the importance of guaranteeing their adequate operation and recovery. These methodologies have proved to be useful for understanding and modeling the operation and recovery of systems of interdependent networks; however, these methodologies tend to be limited and often not suitable for general planning and recovery applications, as they often assume that the characteristics of the studied systems are deterministic and static (Galindo and Batta, 2013). Thus, for general applications, it is important to develop models that can quantify and optimize the resilience of interdependent systems, while considering the uncertainty associated with the system properties.

The uncertainty related to the properties of a system can be associated with several different aspects, such as undetermined demand of goods and services that changes in time, and variable operation and recovery costs caused by unknown availability to labor and materials, among others. To model this source of uncertainty, in addition to using sampling-based approaches (Helton and Davis, 2003; Poss, 2011), some of the most relevant and widely used methodologies include embedded-uncertainty optimization techniques. In the context of optimizing a networked system, these embedded-uncertainty techniques are ofter used to determine optimal design strategies that provide analytical guarantees on the associated system's performance, either by enforcing a given performance range or by maximizing the expected performance.

Even though pure sampling-based approaches could be used to understand and quantify the effects of following an established recovery plan should a disaster occur, they are often not practical to determine which recovery strategy maximizes the overall resilience of the system, since the associated uncertainty is usually modeled externally to the optimization models used. In contrast, embedded-uncertainty optimization approaches, which include robust (Bertsimas and Sim, 2003; Mitra et al., 2005; Ukkusuri et al., 2007; Beyer and Sendhoff, 2007) and stochastic optimization (Shapiro, 2007; Marti et al., 2008; Chen et al., 2010; Hong et al., 2015), allow the explicit inclusion of stochastic parameters and variables in the optimization process (Sahinidis, 2004). In particular, we want to focus on maximizing the expected performance of a system of interdependent networks, while considering the uncertainty associated with the system properties. Thus, we propose a stochastic-optimization mathematical formulation as described next.

4.2.1 The Stochastic INDP model (sINDP)

Particularly, the proposed td-INDP formulation modeled the functionality and the damage state of each element separately, to allow for undamaged elements to be not functional, which constitutes a realistic characteristic caused by the existing element interdependencies within and between networks. Even though this formulation proved to be able to assess multiple desirable characteristics of realistic infrastructure recovery and resilience, such as being able to model diverse types of interdependencies (Rinaldi et al., 2001), the td-INDP assumed that all its parameters were deterministic. Even though this is a common assumption in Operations Research (OR) models for studying disaster operations management and dynamics –in order to keep the models tractable–, in real scenarios, infrastructure systems are subject to uncertainty (Galindo and Batta, 2013). In particular, it is not realistic to assume that parameters such as demands and resources are static (constant in time) and known, thus it is important to develop models that consider uncertainty and allow for time-dependent parameters (Galindo and Batta, 2013), while being able to account for interdependencies and operational constraints.

In order to address these concerns, we propose the Stochastic INDP (sINDP), as the mathematical model that focuses on finding the least-cost time-dependent recovery strategy of a partially destroyed system of interdependent infrastructure networks, subject to budget, resources, and operational constraints, while considering uncertainty associated with demands and resources in the system (González et al., 2017c).

sINDP Formulation

As with the td-INDP model, the proposed sINDP is an MIP formulation that uses as input the information associated with a partially destroyed system of interconnected infrastructure networks (costs, capacities, damaged components, etc.), and returns the recovery process that maximizes the resilience of the system. Thus, the sINDP formulation is based on the same sets used by the td-INDP. These sets include: $\mathcal{L}$, the set of commodities that flow through the system; $\mathcal{K}$, the set of studied networks; $\mathcal{N}$ and $\mathcal{A}$, defined as the sets of nodes and arcs, respectively, that compose the system of networks; $\mathcal{N}_k$ and $\mathcal{A}_k$, which correspond to the sets of nodes and arcs, respectively, that belong to network $k \in \mathcal{K}$; $\mathcal{N}'_k$ and $\mathcal{A}'_k$, the sets of damaged nodes and arcs, respectively, in network $k \in \mathcal{K}$; $\mathcal{L}_k$, the set of commodities that flow through network $k \in \mathcal{K}$; $\mathcal{T}$, defined as the set of periods that compose the recovery time horizon; $\mathcal{S}$, the set of geographical spaces that contain the studied networks; and, $\mathcal{R}$, the set of resources used for the recovery process. Additionally, in order to model the uncertainty associated with parameters such as demand and supply of commodities, availability of resources, and resource utilization during the recovery process, the proposed sINDP also assumes the existence of a finite set of representative scenarios, denominated Ω.

The parameters used by the proposed sINDP also mirror the ones associated with the td-INDP model. However, while some of the sINDP parameters are considered fixed (as in the td-INDP), other parameters are now stochastic, and will dependent on the specific scenario $\omega \in \Omega$. The fixed parameters include: g_{st}, the cost of preparing geographical space s at time t; q_{ikt} and f_{ijkt}, which correspond to the cost of recovering node i or arc (i, j), respectively, in network k at time t; u_{ijkt}, which is the flow capacity of arc (i, j) in network k at time t; β_{ikst} and α_{ijkst}, which indicate if repairing node i or arc (i, j), respectively, requires preparing space s, in network k at time t; and, $\gamma_{ijk\tilde{k}t}$, which indicates if at time t, node i in network k depends on node j in network $\tilde{k} \in \mathcal{K}$. Also, for each scenario $\omega \in \Omega$, the stochastic parameters include: $c_{ijklt\omega}$, the commodity l unitary flow cost through arc (i, j) in network k at time t; $v_{rt\omega}$, the available amount of resource r at time t; $p_{ikrt\omega}$ and $h_{ijkrt\omega}$, the amount of resource r required to recovering node i or arc (i, j), respectively, in network k at time t; $M^{+}_{iklt\omega}$ and $M^{-}_{iklt\omega}$, the cost of excess or lack of supply, respectively, of commodity l in node i in network k at time t; and, $b_{iklt\omega}$, the demand/supply of commodity l in node i in network k at time t. Additionally, P_ω is defined as the probability of occurrence (or more generally, the weight) associated with each scenario $\omega \in \Omega$.

The sINDP decision variables are also analogous to the ones described in the td-INDP. However, as with the parameters discussed above, some variables depend on the scenario $\omega \in \Omega$, while others do not. On one hand, the variables that are not associated with the set of scenarios, are the ones that indicate a recovery strategy, i.e., the variables that indicate when and which elements should be recovered and which geographical spaces should be prepared. These variables are: Δw_{ikt} and Δy_{ijkt}, binary variables to indicate if node i or arc(i, j), respectively, is reconstructed in network k at time t; and, Δz_{st}, which is a binary variable that indicates if space s has to be prepared at time t. On the other hand, the variables that depend of each scenario $\omega \in \Omega$ are the ones associated with the functionality and operation of each network. These variables are: $x_{ijklt\omega}$, flow of commodity l through arc (i, j) in network k at time t; $w_{ikt\omega}$ and $y_{ijkt\omega}$, the variables that indicate the functionality level of node i or arc (i, j), respectively, in network k at time t; and, $\delta^{+}_{iklt\omega}$ and $\delta^{-}_{iklt\omega}$, the excess or lack of supply, respectively, associated with commodity l in node i in network k at time t.

Based on the previous description of the variables and parameters involved, the proposed sINDP MIP model is described as follows:

minimize

$$\begin{aligned}
&\sum_{t\in\mathcal{T}|t>0}\left(\sum_{s\in\mathcal{S}} g_{st}\Delta z_{st} + \sum_{k\in\mathcal{K}}\left(\sum_{(i,j)\in\mathcal{A}'_k} f_{ijkt}\Delta y_{ijkt} + \sum_{i\in\mathcal{N}'_k} q_{ikt}\Delta w_{ikt}\right)\right)\\
&\quad+\sum_{\omega\in\Omega} P_\omega\left\{\sum_{t\in\mathcal{T}}\sum_{k\in\mathcal{K}}\left(\sum_{l\in\mathcal{L}_k}\sum_{i\in\mathcal{N}_k}\left(M^+_{iklt\omega}\delta^+_{ikl t\omega} + M^-_{iklt\omega}\delta^-_{iklt\omega}\right)\right.\right.\\
&\qquad\qquad\left.\left.+\sum_{l\in\mathcal{L}_k}\sum_{(i,j)\in\mathcal{A}_k} c_{ijklt\omega}x_{ijklt\omega}\right)\right\}
\end{aligned}\tag{4.1a}$$

subject to,

$$\sum_{j:(i,j)\in\mathcal{A}_k} x_{ijklt\omega} - \sum_{j:(j,i)\in\mathcal{A}_k} x_{jiklt\omega} = b_{iklt\omega} - \delta^+_{iklt\omega} + \delta^-_{iklt\omega},\quad \forall k\in\mathcal{K},\forall i\in\mathcal{N}_k,\forall l\in\mathcal{L}_k,\forall t\in\mathcal{T},\forall\omega\in\Omega,\tag{4.1b}$$

$$\sum_{l\in\mathcal{L}_k} x_{ijklt\omega} \le u_{ijkt}w_{ikt\omega},\quad \forall k\in\mathcal{K},\forall(i,j)\in\mathcal{A}_k,\forall t\in\mathcal{T},\forall\omega\in\Omega,\tag{4.1c}$$

$$\sum_{l\in\mathcal{L}_k} x_{ijklt\omega} \le u_{ijkt}w_{jkt\omega},\quad \forall k\in\mathcal{K},\forall(i,j)\in\mathcal{A}_k,\forall t\in\mathcal{T},\forall\omega\in\Omega,\tag{4.1d}$$

$$\sum_{l\in\mathcal{L}_k} x_{ijklt\omega} \le u_{ijkt}y_{ijkt\omega},\quad \forall k\in\mathcal{K},\forall(i,j)\in\mathcal{A}'_k,\forall t\in\mathcal{T},\forall\omega\in\Omega,\tag{4.1e}$$

$$\sum_{i\in\mathcal{N}_k} w_{ikt\omega}\gamma_{ijk\tilde{k}t} \ge w_{j\tilde{k}t\omega},\quad \forall k,\tilde{k}\in\mathcal{K},\forall j\in\mathcal{N}_{\tilde{k}},\forall t\in\mathcal{T},\forall\omega\in\Omega,\tag{4.1f}$$

$$w_{ik0\omega} = 0,\quad \forall k\in\mathcal{K},\forall i\in\mathcal{N}'_k,\forall\omega\in\Omega,\tag{4.1g}$$

$$y_{ijk0\omega} = 0,\quad \forall k\in\mathcal{K},\forall(i,j)\in\mathcal{A}'_k,\forall\omega\in\Omega,\tag{4.1h}$$

$$w_{ikt\omega} \le \sum_{\tilde{t}=1}^{t}\Delta w_{ik\tilde{t}},\quad \forall k\in\mathcal{K},\forall i\in\mathcal{N}'_k,\forall t\in\mathcal{T}\mid t>0,\forall\omega\in\Omega,\tag{4.1i}$$

$$y_{ijkt\omega} \le \sum_{\tilde{t}=1}^{t}\Delta y_{ijk\tilde{t}},\quad \forall k\in\mathcal{K},\forall(i,j)\in\mathcal{A}'_k,\forall t\in\mathcal{T}\mid t>0,\forall\omega\in\Omega,\tag{4.1j}$$

$$\sum_{k\in\mathcal{K}}\left(\sum_{(i,j)\in\mathcal{A}'_k} h_{ijkrt\omega}\Delta y_{ijkt} + \sum_{i\in\mathcal{N}'_k} p_{ikrt\omega}\Delta w_{ikt}\right) \le v_{rt\omega},\quad \forall r\in\mathcal{R},\forall t\in\mathcal{T}\mid t>0,\forall\omega\in\Omega,\tag{4.1k}$$

$$\Delta w_{ikt}\alpha_{ikst} \leq \Delta z_{st}, \quad \forall k \in \mathcal{K}, \forall i \in \mathcal{N}'_k, \forall s \in \mathcal{S}, \forall t \in \mathcal{T} \mid t > 0, \tag{4.1l}$$
$$\Delta y_{ijkt}\beta_{ijkst} \leq \Delta z_{st}, \quad \forall k \in \mathcal{K}, \forall (i,j) \in \mathcal{A}'_k, \forall s \in \mathcal{S}, \forall t \in \mathcal{T} \mid t > 0, \tag{4.1m}$$
$$\delta^+_{ikltω} \geq 0, \quad \forall k \in \mathcal{K}, \forall i \in \mathcal{N}_k, \forall l \in \mathcal{L}_k, \forall t \in \mathcal{T}, \forall \omega \in \Omega, \tag{4.1n}$$
$$\delta^-_{ikltω} \geq 0, \quad \forall k \in \mathcal{K}, \forall i \in \mathcal{N}_k, \forall l \in \mathcal{L}_k, \forall t \in \mathcal{T}, \forall \omega \in \Omega, \tag{4.1o}$$
$$x_{ijkltω} \geq 0, \quad \forall k \in \mathcal{K}, \quad \forall (i,j) \in \mathcal{A}_k, \forall l \in \mathcal{L}_k, \forall t \in \mathcal{T}, \forall \omega \in \Omega, \tag{4.1p}$$
$$w_{iktω} \geq 0, \quad \forall k \in \mathcal{K}, \forall i \in \mathcal{N}_k, \forall t \in \mathcal{T}, \forall \omega \in \Omega \tag{4.1q}$$
$$y_{ijktω} \geq 0, \quad \forall k \in \mathcal{K}, \forall (i,j) \in \mathcal{A}'_k, \forall t \in \mathcal{T}, \forall \omega \in \Omega, \tag{4.1r}$$
$$\Delta w_{ikt} \in \{0,1\}, \quad \forall k \in \mathcal{K}, \forall i \in \mathcal{N}'_k, \forall t \in \mathcal{T} \mid t > 0 \tag{4.1s}$$
$$\Delta y_{ijkt} \in \{0,1\}, \quad \forall k \in \mathcal{K}, \forall (i,j) \in \mathcal{A}'_k, \forall t \in \mathcal{T} \mid t > 0, \tag{4.1t}$$
$$\Delta z_{st} \in \{0,1\}, \quad \forall s \in \mathcal{S}, \forall t \in \mathcal{T} \mid t > 0. \tag{4.1u}$$

The logic behind the proposed objective function (4.1a) is analogous to the one associated with the td-INDP's objective function (2.1a) presented in Section 2.2.2. However, the objective function (4.1a) now focuses on optimizing the overall expected recovery costs considering all scenarios $\omega \in \Omega$, as opposed to only optimizing for a single scenario. Similarly, the sINDP constraints (4.1b)-(4.1m) are also analogous to the td-INDP constraints (2.1b)-(2.1m). However, the proposed sINDP assumes that multiple parameters are subject to uncertainty, including the costs of unitary flow, the availability of resources, the demand and supply of commodities, the resource utilization necessary to perform a recovery job, and the costs of excess supply and unsatisfied demand. Given this, the sINDP formulation constructs a full set of td-INDP-like constraints for each scenario $\omega \in \Omega$, to ensure that the proposed recovery strategy guarantees that all operation and functionality constraints are complied for each individual scenario studied. Considering this, note that the sINDP consists of multiple coupled td-INDP-like constraints. Thus, building upon the unique properties of the td-INDP (as shown in Section 3.2.2), it is expected that the proposed sINDP formulation has a structure suitable for decomposition techniques, which is detailed as follows.

4.2.2 Structure of the sINDP

First, let us note that the proposed sINDP formulation can be described as a two-stage stochastic problem, where the first stage is associated with determining a suitable recovery strategy for a damaged system, and the second stage is related to evaluating the operation and performance of the system for each scenario $\omega \in \Omega$ (if the suggested recovery strategy takes place) (González et al., 2017c). In general, such two-stage stochastic problems can be associated with the following (extensive) form:

$$\text{minimize} \quad \tilde{c}^T\tilde{x} + \sum_{\omega\in\Omega} P_\omega Q_\omega^T \tilde{y}_\omega \tag{4.2a}$$

subject to,

$$A\tilde{x} = B \tag{4.2b}$$

$$T_\omega\tilde{x} + W\tilde{y}_\omega = \tilde{h}_\omega, \quad \omega \in \Omega; \tag{4.2c}$$

$$\tilde{x} \in \{0,1\}, \tag{4.2d}$$

$$\tilde{y}_\omega \geq 0, \quad \omega \in \Omega. \tag{4.2e}$$

For the particular case of the sINDP formulation, the sINDP binary recovery variables are related to the first-stage stochastic variables, as described as follows:

$$\tilde{x} \equiv \{\Delta z_{st}, \Delta w_{ikt}, \Delta y_{ijkt}\} \tag{4.3}$$

Similarly, the sINDP operational variables, associated with the functionality of the elements and the operation of the system, are related to the second-stage variables, as described below:

$$\tilde{y}_\omega \equiv \{\delta^+_{ikl t\omega}, \delta^-_{ikl t\omega}, x_{ijkl t\omega}, w_{ikt\omega}, y_{ijkt\omega}\} \tag{4.4}$$

Considering this relation between the sINDP variables and the variables of the two-stage program in extensive form, the associated first- and second-stage objective functions are related as follows:

$$\tilde{c}^T\tilde{x} \equiv \sum_{t\in\mathcal{T}|t>0}\left(\sum_{s\in\mathcal{S}} g_{st}\Delta z_{st} + \sum_{k\in\mathcal{K}}\left(\sum_{(i,j)\in\mathcal{A}'_k} f_{ijkt}\Delta y_{ijkt} + \sum_{i\in\mathcal{N}'_k} q_{ikt}\Delta w_{ikt}\right)\right) \tag{4.5}$$

and

$$Q_\omega^T \tilde{y}_\omega \equiv \sum_{t\in\mathcal{T}} \sum_{k\in\mathcal{K}} \left(\sum_{l\in\mathcal{L}_k} \sum_{i\in\mathcal{N}_k} \left(M^+_{ikltω}\delta^+_{ikltω} + M^-_{ikltω}\delta^-_{ikltω} \right) + \sum_{l\in\mathcal{L}_k} \sum_{(i,j)\in\mathcal{A}_k} c_{ijkltω} x_{ijkltω} \right) \tag{4.6}$$

Finally, regarding the constraints, constraints (4.2b) correspond to constraints (4.1l)-(4.1m), and constraints (4.2c) correspond to constraints (4.1b)-(4.1k).

Following these equivalencies, Figure 4.5 describes a typical representation of the sINDP structure, which shows its particular block structure. As described in Section 3.2.2, having a mathematical formulation with a structure such as the one exhibited by the sINDP, often indicates that the associated solution times can be improved by applying diverse decomposition techniques, such as Benders and Dantzig-Wolfe decomposition. In particular, two-stage stochastic programs such as the one described by equations (4.2a)-(4.2e) have been shown to be specially susceptible to a decomposition strategy denominated L-shaped method (Slyke and Wets, 1969; Miller-Hooks et al., 2012; Faturechi et al., 2014), which could be seen as an extension of Benders decomposition, but that exploits the fact that the coefficients in the second-stage represent a fully block-diagonal structure, without coupling constraints. Moreover, note that for the particular case of the proposed sINDP formulation, the matrix W has a special structure on its own, caused by the co-existence of multiple interdependent networks and multiple coupled recovery periods. In particular, each W matrix is associated with a td-INDP-like set of constraints; thus, W could be further decomposed as described in Section 3.2.2.

The proposed sINDP fits a two-stage stochastic problem, since we assumed that the recovery strategy is determined immediately after the disaster occurs, and once the recovery process starts, the recovery strategy is considered fixed, and only the system's operation is optimized for each period, depending on the observed demands,

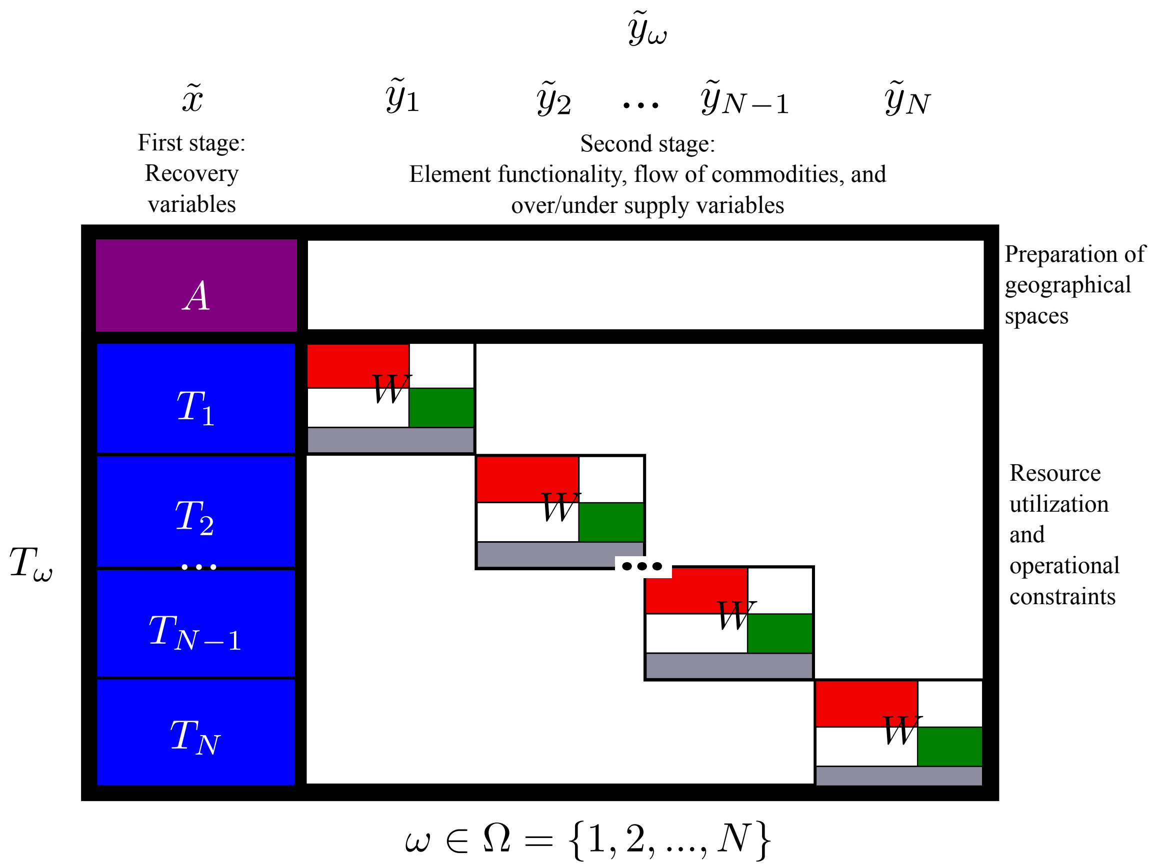


Figure 4.5 : Decomposition of the sINDP MIP model as a two-stage stochastic problem (González et al., 2017c)

costs, etc. In other words, the decisions associated with the first stage of the sINDP –planning the recovery strategy that will be implemented in the subsequent periods– would be taken between period 0 (when the disaster occurs) and period 1 (when the recovery process starts), while the decisions associated with the second stage –deciding the flows in the system and supply of demands, considering the elements that are recovered at each period– would be taken between period 1 and the last period in the planning horizon. However, in some instances, we may assume that planning the recovery strategy involves a sequence of decisions, that are updated at each period in response to realizations of outcomes that were initially unknown. In this context, the resultant sINDP model could be expanded to a multi-stage stochastic problem. For this case, at each period t the decision maker would have full knowledge of the damage states of all elements in the system, and will focus on determining the elements to be recovered at period $t+1$, taking into account that the system properties (demands, resources, costs, etc) are known up to period t, but unknown for periods $t+1$ and above. Nevertheless, it is important to note that in this case the probability distributions for the random variables associated with time $t+1$ will depend on the realizations observed for these variables at time t. Thus, it can be seen how the complexity of such optimization problem would quickly increase with the number of periods, reducing its tractability for large planning horizons.

4.2.3 sINDP illustrative example

To illustrate the use of the proposed sINDP model, we present a study on the system of utility networks described in Section 2.3.2. For this example, we simulated different earthquakes, again with epicenter at $35.3° N$ and $90.3° W$ (as described in Adachi and Ellingwood (2009, 2010)) and magnitudes $M_w \in \{6, 6.5, 7\}$. For each earthquake-related disaster scenario, we applied the sINDP to find the associated optimal recovery strategies, using planning horizons that span from 1 to 10 recovery periods.

In this computational experiment, we studied the effects of considering the uncertainty associated with unknown demands. In particular, for each disaster realization we considered 100 different demand scenarios. These demand scenarios were randomly generated, assuming that the demands had a uniform distribution within an interval of $\pm 20\%$ the estimated expected value of each demand. The estimated expected demands, flow and recovery costs, as well as network flow capacities, are consistent with the deterministic ones used in used in Section 2.3.2 (González et al., 2016b).

In order to generate a representative set of demand scenarios that are adequately spread over the input space, we used Latin Hypercube Sampling (Helton and Davis, 2003; Iman, 2008) while maximizing the minimimum distance between each pair of sample points. To solve the sINDP formulation for each disaster scenario, we used JuMP (Julia modeling language for Mathematical Optimization) (Bezanson et al., 2017; Dunning et al., 2017), in conjunction with diverse leading commercial optimizers, to show that the proposed sINDP formulation can be solved efficiently in readily-available software used in practice. In particular, we solved the proposed problems using Gurobi V7.0.1, Xpress-MP V7.9, and CPLEX V12.7, using their default configurations[†]. Additionally, we also solved the proposed problems using L-shaped decomposition in CPLEX V12.7. The solution times associated with each of these four cases (denoted as C1, C2, C3a, and C3b, respectively) are shown in Table 4.1.

As expected, Table 4.1 shows that the larger the earthquake magnitude, the larger the solving times. Also, Table 4.1 shows that the solving instances that use CPLEX with L-Shaped decomposition result, on average, in faster solving times. In particular, using CPLEX with L-Shaped decomposition was 41% faster than using CPLEX with its default configuration, showing the usefulness of decomposition approaches to solving the sINDP. Figure 4.6 shows these sINDP solving times as a function of the periods in the planning horizon. This Figure shows that, as observed for the td-INDP (Section 3.1), the sINDP solving times grow exponentially with the cardinality of $\mathcal{T}$, independently of the optimizer or the solution approach.

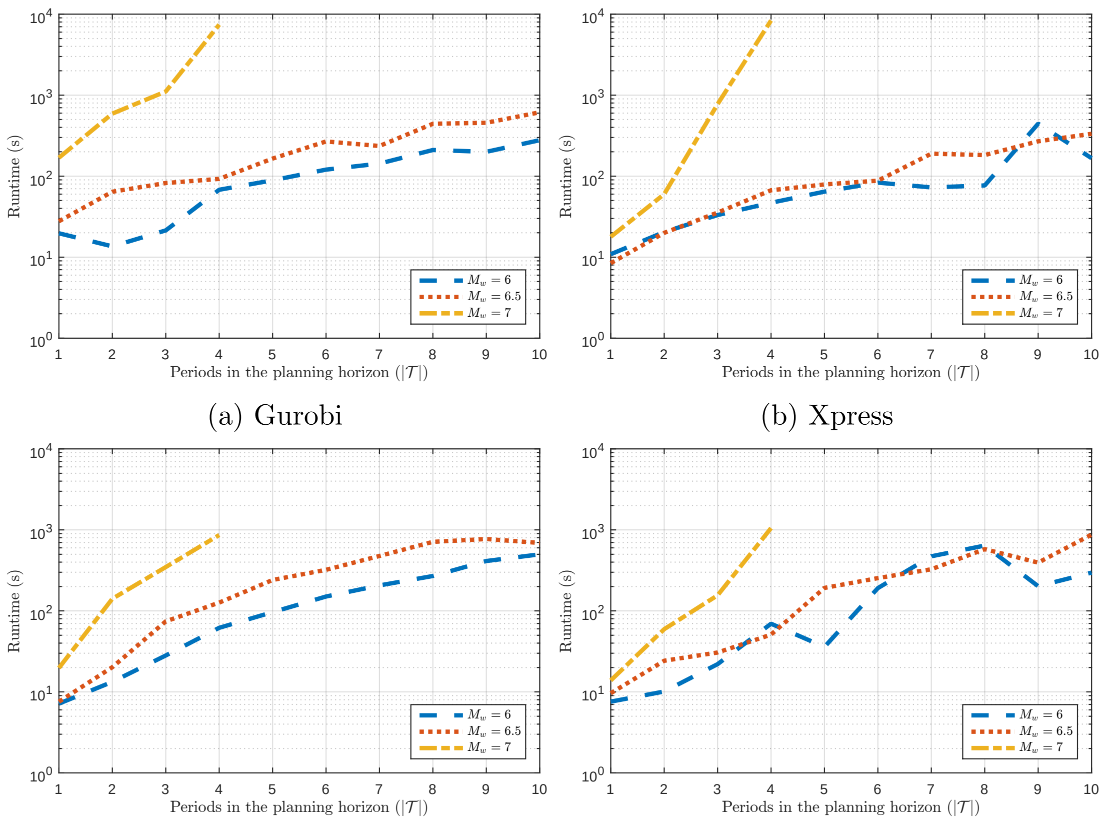
To illustrate the optimal recovery strategies found for each disaster realization (ω), Figure 4.7 shows the evolution of the expected performance associated with each earthquake magnitude studied. The measured performance corresponds to the ratio

[†]For these computational experiments, we used a PC with Windows 7 Enterprise Service Pack 1 of 64 bits, an Intel Core i7-4785T CPU @ 2.20GHz, and 16 GB of RAM

Table 4.1 : sINDP solution times for different earthquake magnitudes and time horizons, using different leading optimizers (González et al., 2017c)

		Runtime (s)				Speedup		
Magnitude	**Periods** ($\lvert\mathcal{T}\rvert$)	**C1: Gurobi**	**C2: Xpress**	**C3a: Cplex**	**C3b: Cplex (L-Shaped)**	**C1/C3b**	**C2/C3b**	**C3a/C3b**
6	1	19.72	10.87	7.16	7.58	2.60	1.43	0.95
6	2	13.59	20.41	13.30	10.11	1.34	2.02	1.32
6	3	21.40	33.12	28.15	22.01	0.97	1.50	1.28
6	4	67.93	46.92	61.81	69.44	0.98	0.68	0.89
6	5	88.85	64.40	96.48	35.90	2.47	1.79	2.69
6	6	120.42	83.20	150.47	191.12	0.63	0.44	0.79
6	7	142.95	72.59	206.31	472.08	0.30	0.15	0.44
6	8	210.31	76.73	269.63	640.63	0.33	0.12	0.42
6	9	199.67	444.49	412.81	201.80	0.99	2.20	2.05
6	10	277.97	166.66	499.11	296.74	0.94	0.56	1.68
6.5	1	27.75	8.43	7.56	9.63	2.88	0.88	0.79
6.5	2	64.20	19.95	20.19	24.23	2.65	0.82	0.83
6.5	3	82.21	35.51	74.28	30.64	2.68	1.16	2.42
6.5	4	92.53	66.97	126.52	50.95	1.82	1.31	2.48
6.5	5	164.85	79.02	240.84	192.15	0.86	0.41	1.25
6.5	6	268.23	87.95	321.42	253.88	1.06	0.35	1.27
6.5	7	235.70	189.43	476.37	326.54	0.72	0.58	1.46
6.5	8	443.37	182.12	711.96	578.20	0.77	0.31	1.23
6.5	9	456.15	269.25	771.56	394.54	1.16	0.68	1.96
6.5	10	614.34	333.14	690.20	874.40	0.70	0.38	0.79
7	1	168.83	17.84	19.75	13.85	12.19	1.29	1.43
7	2	591.49	59.96	141.17	59.45	9.95	1.01	2.37
7	3	1109.54	775.26	347.60	155.74	7.12	4.98	2.23
7	4	7426.07	8347.17	864.05	1051.90	7.06	7.94	0.82
					Mean	2.63	1.37	1.41
					Mean (<1)	0.74	0.49	0.75
					Mean (>1)	4.23	2.42	1.81

between the total satisfied demands and the total demands in the system of networks.



(a) Gurobi

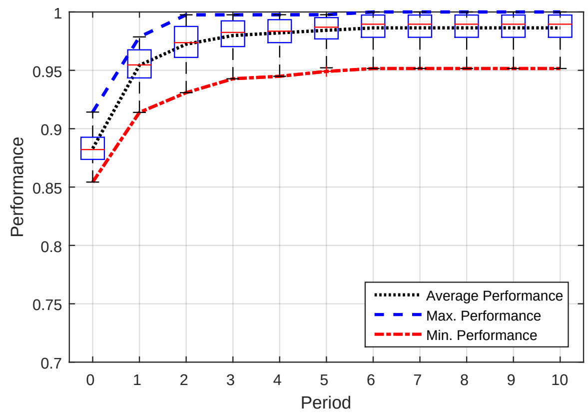
(b) Xpress

(c) Cplex

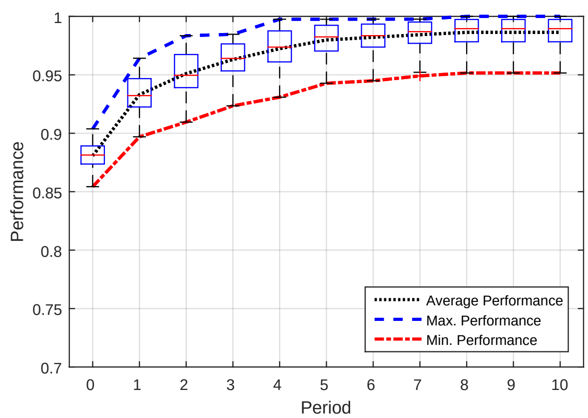
(d) Cplex (Benders)

Figure 4.6 : Solution times for the sINDP as a function of $|\mathcal{T}|$, using different optimizers (González et al., 2017c)

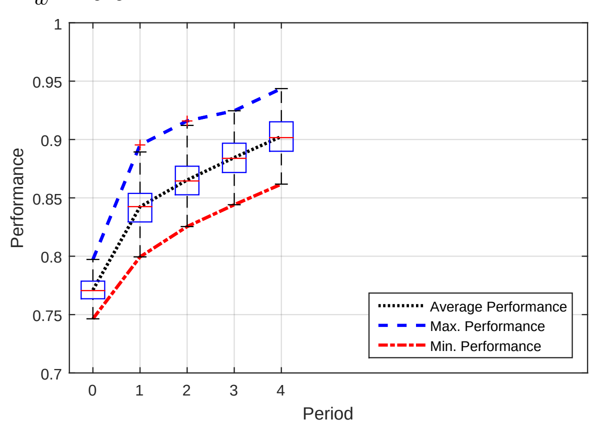
This Figure shows that the optimal expected recovery tends to always increase, with a slope that is non-negative and monotonically decreasing, as observed for the deterministic td-INDP (Section 2.3.2). This serves as an indicator that heuristic approaches such that the iINDP may be successfully applied to solving the sINDP. As expected, Figure 4.7 shows that the larger the initial damage, the worse the expected performance. Moreover, Figure 4.7 also shows a box-plot representation of the performances associated with all individual demand scenarios. Note that not all scenarios reach a full recovery, even after the recovery process has finished. This is



(a) Recovery across demand scenarios, for $M_w = 6$



(b) Recovery across demand scenarios, for $M_w = 6.5$



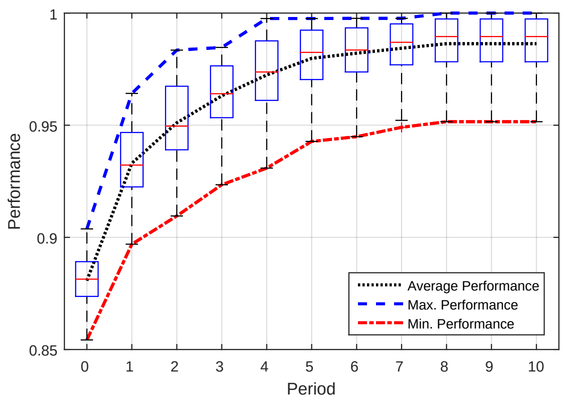
(c) Recovery across demand scenarios, for $M_w = 7$

Figure 4.7 : Performance recovery for $M_w \in \{6, 6.5, 7\}$ (González et al., 2017c)

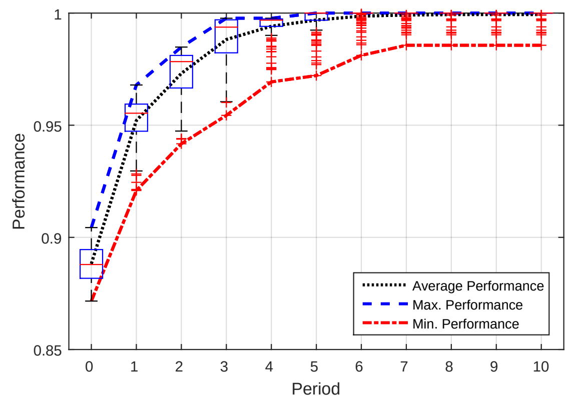
caused by the fact that in some scenarios, the total demand was higher than the total supply capacity. Such cases can occur since we assumed that the total supply capacity was perfectly balanced with the total average demand.

In order to gain some insight on how the total supply capacity influences the performance of the studied system of networks, we also analyzed four different supply capacity cases in which the total supply capacity had a surplus with respect to the total average demand: supply surplus of 0% (i.e., the case where both supply and average demand are perfectly balanced); supply surplus of 5%; supply surplus of 10%; and, supply surplus of 20%. To illustrate the effects of considering a supply surplus –which is customary in practical systems, where reserve capacity is expected for reliability and management of peak demands– on the expected performance recovery, Figure 4.8 shows the expected performance recovery for the four supply surplus cases, assuming an earthquake magnitude $M_w = 6.5$. As expected, it can be seen that the more the considered supply surplus, the higher the associated expected performance at each period. Moreover, note that the variability associated with the performance of each individual demand scenario is also reduced when considering greater supply capacities. In fact, for the cases modeling a surplus of 10% and 20%, the performance could be fully recovered for all studied demand scenarios.

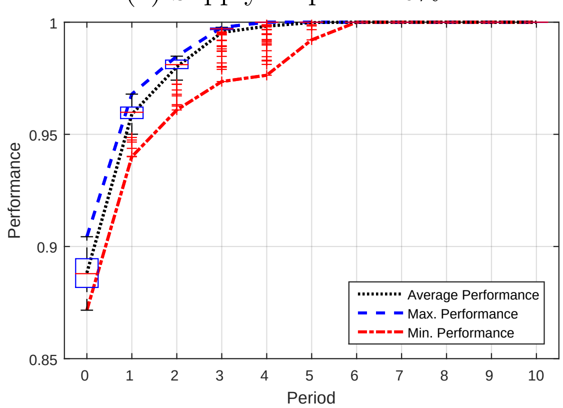
However, considering additional supply capacity may imply increasing the amount of undelivered commodities. Figure 4.9 depicts the undelivered supply associated with the previously described disaster realization. As expected, this Figure shows that the amount of undelivered commodities increases dramatically with the magnitude of the supply surplus. This analysis would offer decision makers and stakeholders important information about the possible costs associated with a given disaster, related to both having unmet demands and having undelivered commodities, depending on the level of reserve capacity. To offer additional insights of the trade-offs between pre-event



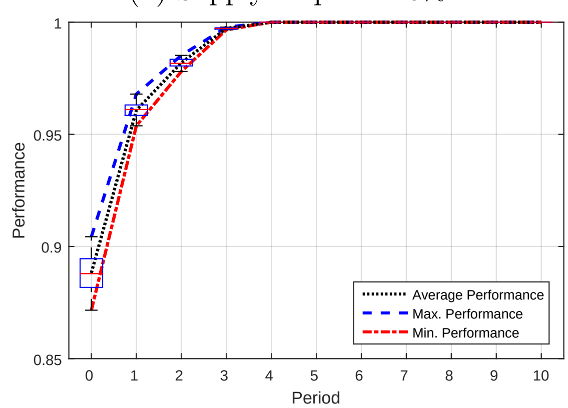
(a) Supply surplus = 0%



(b) Supply surplus = 5%

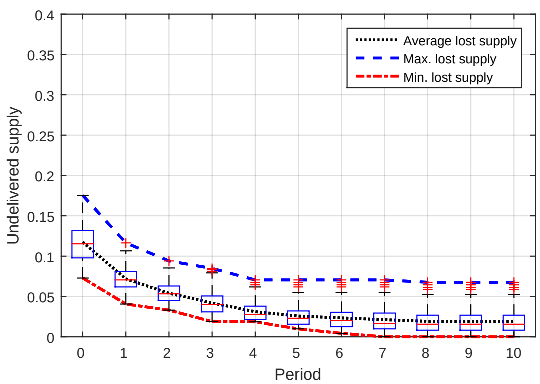


(c) Supply surplus = 10%

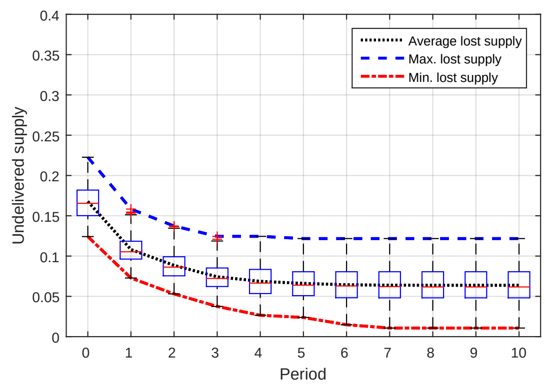


(d) Supply surplus = 20%

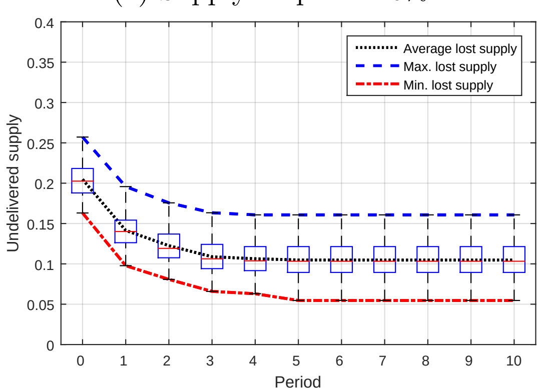
Figure 4.8 : System perfomance for each recovery period, consistent with an earthquake of magnitude $M_w = 6.5$ (González et al., 2017c)



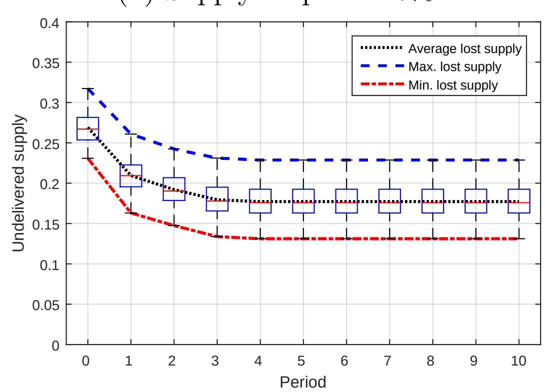
(a) Supply surplus = 0%



(b) Supply surplus = 5%



(c) Supply surplus = 10%



(d) Supply surplus = 20%

Figure 4.9 : Undelivered supply as a function of time, consistent with an earthquake of magnitude $M_w = 6.5$ (González et al., 2017c)

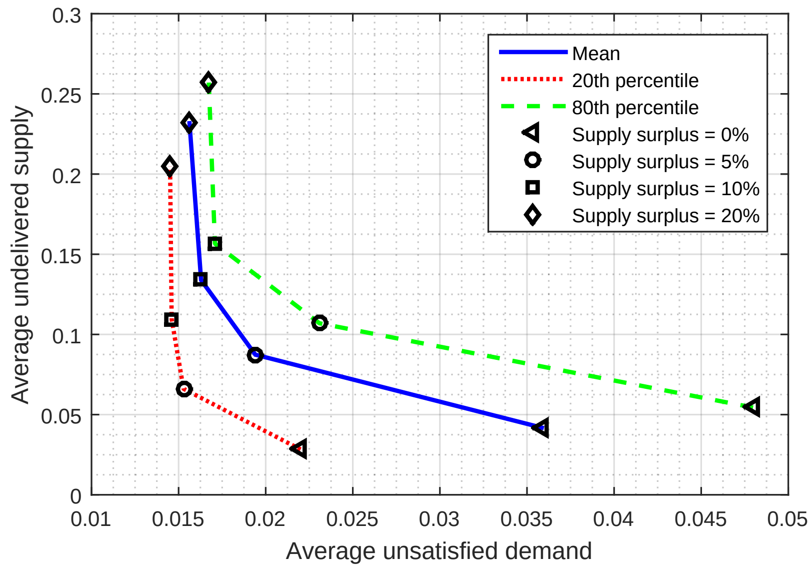
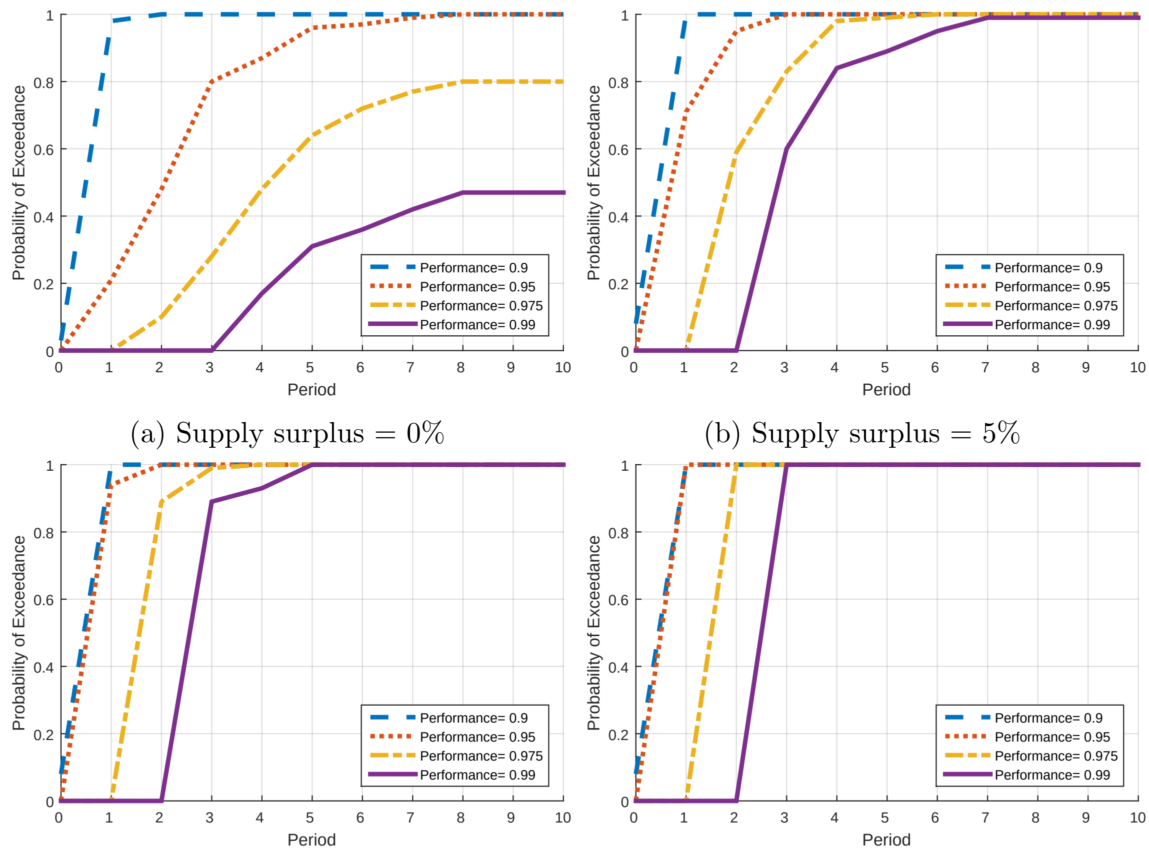


Figure 4.10 : Trade-off between undelivered supply and unsatisfied demand, for diverse supply-surplus values (González et al., 2017c)

decisions and post-event performance recovery, Figure 4.10 shows how sensitive is the average undelivered supply as a function of the average unsatisfied demand. This Figure shows that the performance of the system is highly sensitive to changes in the supply capacity, when the supply surplus is below 10%. However, designing the system with a supply surplus higher than 10%would have almost no effects on the average performance of the system, but would highly impact the undelivered supply.

Another use of the proposed sINDP model is associated with estimating the probability of exceeding a certain performance level, for each recovery period. These exceedance plots are often used to depict and quantify the risk associated with damaging events, such as earthquakes, floods, and hurricanes, among others. Figure 4.11 shows the probability of exceedance of different performance levels as a function of the recovery period, consistent with an earthquake of magnitude $M_w = 6.5$. This Figure shows that the risk related to the modeled earthquake is reduced with the size of the supply surplus considered. For example, on one hand, Figure 4.11a shows that if the supply and the total average demand are perfectly balanced, after only one period a recovery of 90% of the performance is almost certain, whereas a recovery of 95% of the performance would only have about 20% probability of occurrence. On the other hand, Figure 4.11d shows that for a supply surplus of 20%, recovering the system to more than 95% performance is almost certain after just one period.



(a) Supply surplus = 0%

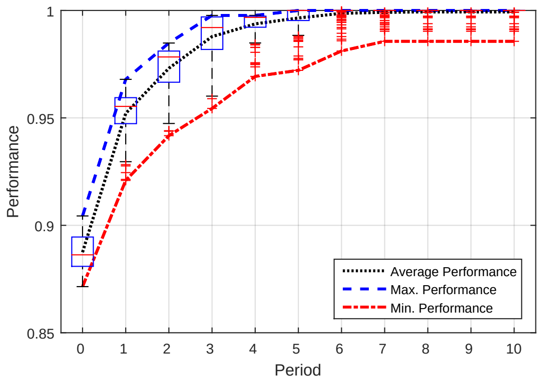
(b) Supply surplus = 5%

(c) Supply surplus = 10%

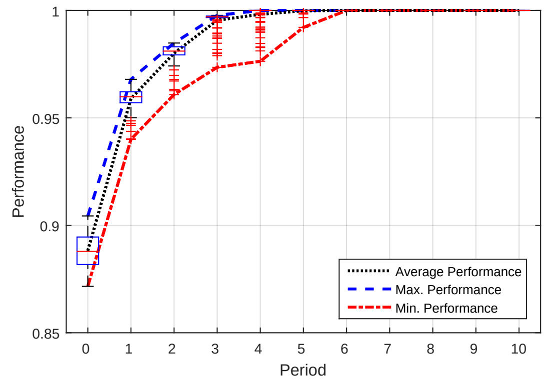
(d) Supply surplus = 20%

Figure 4.11 : Probability of exceedance of different performance levels as a function of the recovery period, consistent with an earthquake of magnitude $M_w = 6.5$ (González et al., 2017c)

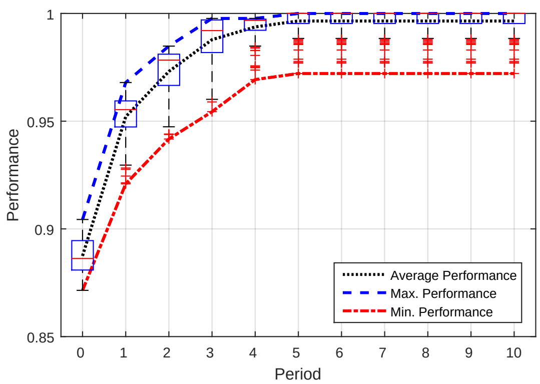
These analyses show that being able to embed uncertainty in the recovery optimization process, enables constructing probabilistic studies that inform not only about the expected recovery, but also about the associated variability. However,



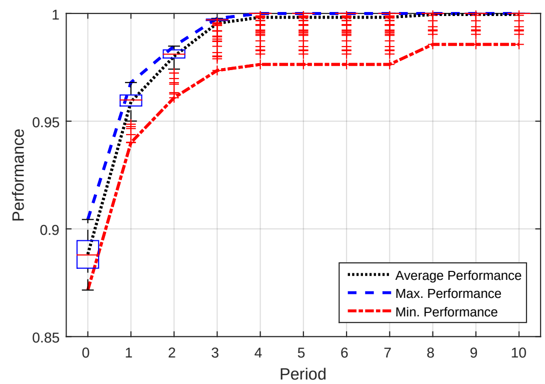
(a) sINDP solutions (Supply surplus = 5%)



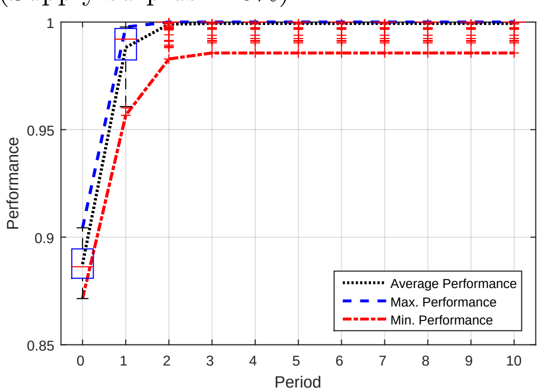
(b) sINDP solution (Supply surplus = 10%)



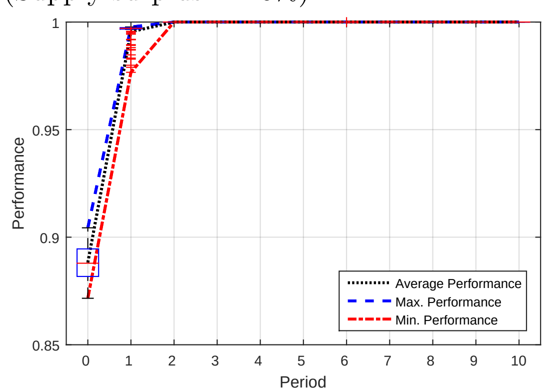
(c) Solutions assuming average demands (Supply surplus = 5%)



(d) Solutions assuming average demands (Supply surplus = 10%)



(e) Solutions assuming perfect Information (Supply surplus = 5%)



(f) Solutions assuming perfect Information (Supply surplus = 10%)

Figure 4.12 : Comparison between three different performance recovery cases (assuming supply surplus of 5% (column 1) and 10% (column 2)): considering all scenarios simultaneously (row 1); using only the expected value of the demands (row 2); and, assuming perfect information (row 3) (González et al., 2017c).

solving the sINDP may be computationally taxing, as it requires finding the optimal recovery process while considering all demand scenarios simultaneously, as opposed to a simulation-based approach where one would solve for each scenario independently. Moreover, it may be the case that considering uncertainty in the system would lead to the same results to a case where perfect information was available. In order to better quantify the advantages of using the proposed sINDP, let us compare three different cases:

- Assuming the use of the proposed sINDP as previously discussed.
- Assuming knowledge about the expected demands only. For this case, first solve the sINDP assuming that all demand scenarios are modeled using only the expected demands. Then, using the obtained recovery strategy, evaluate the performance of each of the 100 different demand scenarios previously described. This case quantifies the value of using the proposed stochastic approach.
- Assuming full knowledge of perfect information. In this case, assume that all information about each demand scenario is fully known before seeking the optimal recovery strategy. This would imply that each demand scenario would represent a deterministic case, and the td-INDP could be used to find its associated optimal recovery strategy (independently on the other demand scenarios). This case quantifies the benefits of having perfect information.

Figure 4.12 depicts the proposed comparison between these three cases, considering a supply surplus of 5% and 10% for the disaster realization related to $M_w = 6.5$. Figures 4.12a and 4.12b show the optimal performance recovery calculated using the sINDP.

Figures 4.12c and 4.12d show the average performance recovery, assuming that the recovery strategy imposed was found using only the expected values of the demands.

Note that the average performance recovery is less when implementing the recovery strategy constructed using only the expected demands. Moreover, note that the resultant performance uncertainty is also much greater, particularly at the end of the recovery horizon. For example, assuming a supply surplus of 10%, if one uses a recovery strategy based only on the expected demands, there would be multiple scenarios in which the performance is not fully recovered from period 6 and beyond, as opposed to the case of the sINDP, where all scenarios are recovered to full performance.

Figures 4.12e and 4.12f show the performance recovery if there was perfect information for each demand scenario. As expected, it can be seen that the average recovery process would take place much faster, since for each studied scenario we would be using its optimal recovery strategy. The shown results indicate that for this particular problem, there would be great value in accurately measuring and forecasting the demands.

4.3 Conclusions

In this Chapter we discuss the importance of considering different sources of uncertainty when modeling and optimizing the resilience of a system of interdependent networks. In particular, we detail two sources of uncertainty that are of relevance when studying realistic scenarios: uncertainty associated with the failure modes of the system, and uncertainty related to the properties of the system.

To address the first source of uncertainty, related to the failure modes of the system given external hazards, we show that models proposed in previous Chapters (such as the td-INDP, and its heuristic approach, the iINDP) could be integrated with simulation-based techniques to perform both pre- and post-event analysis. By doing so, we show that the proposed simulation approach can be used to inform stakeholders and decision makers about system- and element-wise resilience metrics. In particular, Section 4.1 illustrates how the proposed simulation approach can be used to construct relevant resilience metrics associated with the expected failure likelihood, recovery likelihood, and recovery times of each individual component in the system. This Section demonstrates that the proposed resilience metrics could be used to construct importance rankings that are consistent for different disaster levels, thus offering an important tool for mitigation and preparedness.

To address the second source of uncertainty, related to the operational properties of the system, Section 4.2.1 presents a new two-stage stochastic program, denominated the stochastic INDP (sINDP). We show that the proposed sINDP could be used to determine the optimal recovery strategy of a damaged system, while considering uncertainty in multiple parameters such as demand of commodities and resource availability. In particular, Section 4.2.1 presents an illustrative example based on the system of water, power, and gas networks described in Section 2.3.2, where we show the advantages of using the proposed sINDP. For example, we discuss how to use the sINDP to optimize the expected recovery of a damaged system, while studying the effects of increasing the system's supply capacity.

Additionally, we show that the sINDP is susceptible to decomposition strategies, such as the L-shaped method, in order to increase the average solution speeds. Note that there are recent promising developments in decomposition techniques that could be applied in the future to further accelerate the sINDP solving times. For example, Tarvin et al. (2016) proposed an approach that uses enumeration and both serial and parallel computing to exploit the structure of two-stage stochastic problems with binary master problems. Also, note that heuristic approaches, such as the proposed iINDP, may be applied to enhance the sINDP's solving efficiency.

Conceptually, the proposed sINDP is consistent with a two-stage stochastic formulation, since the recourse problem associated with the operation of the system (in the second stage) results in an optimization problem that effectively minimizes the expected costs (or maximizes the expected performance) of the system during the recovery process. However, in some instances, optimizing the expected costs may not be the priority of the decision maker. Other desirable objectives may include ensuring that the performance of the system meets a desired threshold, or that the probability of complying with a set of operational constraints is above a certain level. In order to formulate such optimization problems, additional modeling approaches should be explored, including robust optimization and chance-constrained optimization, among others. For the context of infrastructure systems and their resilience, a stochastic formulation would focus on optimizing the average or expected performance of the system, whereas robust optimization would focus on guaranteeing that the performance in the worst-case scenario is as high as possible.

Thus, in general, selecting the modeling approach to be used depends directly on the specific goals of the decision maker, for which these approaches could be used in isolation or combined. For example, when studying diverse risk measures widely used in practice, such as the Value at Risk (VaR) or the Conditional Value at Risk (CVaR), we may use one or multiple of these modeling approaches simultaneously. If focusing on analyzing the performance of a given system, the VaR associated with a certain confidence value $\hat{\alpha}$ corresponds to the performance level P_{VaR} such that the probability of having a performance below P_{VaR} is $\hat{\alpha}$. Similarly, the CVaR associated with a certain confidence value $\hat{\alpha}$ is defined as the expected performance level calculated considering only performances below the worst $\hat{\alpha}$-percentage value (Bertsimas et al., 2011; Rockafellar, 2007; Di Domenica et al., 2007). Developing analytical and computational approaches that can quantify such risk measures in a context of infrastructure resilience would enable studying how sensitive the pre- and post-event decisions are as a function of the decision makers' risk averseness level, in order to determine adequate mitigation and recovery strategies that align with the needs and constraints of infrastructure operators and stakeholders.

Part III

Beyond the INDP

Chapter 5

Expansions/Applications of the INDP

Previous Chapters have introduced the Interdependent Network Design Problem (INDP), as the problem that focuses on finding the recovery strategy that minimizes the costs associated with reconstruction and operation of a system of interdependent networks. We also proposed diverse approaches to solve the INDP, including the time-dependent INDP (td-INDP), the iterative INDP (iINDP), and the stochastic INDP (sINDP) models. These models have proven useful in finding the optimal recovery strategy for a system of networks, while considering diverse interdependencies between them, as well as realistic operational constraints, such as limited resources and finite capacities. Similarly, we showed how these models could be used to study the resilience of a system, while taking into account diverse sources of uncertainty associated with the system's failure modes and with the system's properties. In this Chapter, we propose further extensions of the INDP models, such that they can address the following additional challenges*:

First, current recovery and resilience models available in academic literature often assume that there is a single entity with complete decision power over all interdependent networks. This may happen in a few selected cases, such as the illustrative example related to the water, power, and gas networks in Shelby County, TN., since all these

*This Chapter is based on the ideas and contents presented in González et al. (2017a) and Smith et al. (2016). These publications are part of the collaborative Multidisciplinary University Research Initiative (MURI) work with the Ph.D. student Andrew Smith, Professor Raissa D'Souza, Professor Mehran Mesbahi, and Professor Airlie Chapman. The definitive versions of these papers will be available at www.onlinelibrary.wiley.com and www.blackwell-synergy.com utilities are handled by a single company: Memphis Light, Gas, and Water (MLGW). However, this is often not the case, and in reality there may be multiple decision makers involved in the operation and recovery of the system of systems. Thus, it is important to develop models that enable studying the dynamics of a system of interdependent networks as decentralized or distributed systems.

Second, available models often do not facilitate identifying the particular properties and dynamics related to the recovery and resilience of the system, since they usually only receive input information regarding the system's damage state, and output the associated optimal recovery strategies. Studies such as the one presented in Section 4.1, which analyzed diverse element-wise resilience metrics, provide important insights about the relative importance of individual elements, which could be used for mitigation and preparedness planning. However, these do not permit uncovering the intricate recovery processes and relations between elements, which are product of multiple relevant factors, such as the networks' topologies and their diverse interdependencies. Thus, it is important to propose strategies that allow not only determining efficient recovery strategies for damaged systems of interdependent networks, but also that enable identifying key properties and dynamics from the recovery process.

In this Chapter, we present different multidisciplinary applications developed to address these emerging challenges. In Section 5.1 we discuss the importance of considering decentralized decision making, and present modeling approaches in order to depict more realistic recovery dynamics, associated with human-made and human-managed systems. In Section 5.2 we propose a mathematical framework for system identification oriented to extracting and modeling the main recovery dynamics and processes of a system of interdependent networks.

5.1 Socio-technical constraints / Decentralized optimization

As we have discussed in previous Chapters, there is an important body of work associated with modeling and quantifying the vulnerability and reliability of infrastructure networks. These methodologies have proven useful for analyzing the behavior and dynamics of networked systems, such as transportation, telecommunication, power, water, and other infrastructure networks. However, many of these methodologies are often based on assumptions that may be impractical for realistic scenarios. For example, in Chapter 4 we discussed how several of these methodologies often assume deterministic and static parameters (Galindo and Batta, 2013). Unfortunately, many of these models also assume the existence of a single entity in charge of taking all decisions at a system level. Moreover, these methodologies are often based on centralized optimization approaches, that assume that an all-powerful individual entity not only takes all decisions in the system, but also counts with the information related to all elements in the system, such as their current damage states.

For example, the proposed td-INDP focuses on determining the recovery strategy that minimizes the costs associated with the reconstruction process, the system's operation, and unsupplied demands; the costs being minimized by the td-INDP are related to the total costs of the system of interdependent networks. However, minimizing the total global costs in general is not equivalent to minimizing the cost for each individual infrastructure network. Thus, if each network implements the suggested td-INDP strategy, they may not be minimizing their own particular costs. Even though a recovery strategy that minimizes the global costs may be seen as ideal from the population's and the government's point of view, it is likely that each individual infrastructure network would not follow such a globally-oriented strategy, as it may harm its own particular costs. In contrast, in realistic scenarios, each infrastructure system may tend to take decisions separately, focusing on minimizing their own individual costs.

Additionally, decision makers from each infrastructure system often do not disclose full and detailed information about their networks (or particularly after a disaster, about their damaged components). In such cases, decision makers from each infrastructure network may implement strategies that will not consider their dependence to other networks accurately. Thus, they may end up implementing strategies that not only increase the global costs, but also increase their own individual costs (and reduce performance).

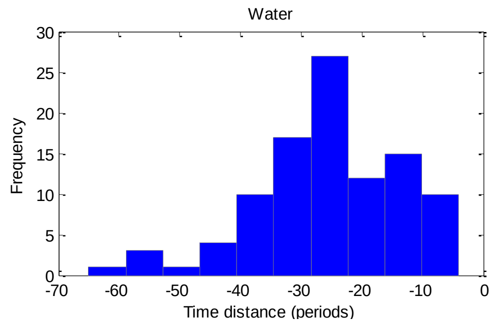
To illustrate such a case, assume that after a certain disaster scenario, each network in the system of interdependent utility networks studied earlier (the water, gas, and power networks from Shelby Count, TN.) designs their own recovery strategies, following an individually-oriented optimization approach that does not consider interdependencies with other networks. Then, it is easy to see that, when following such strategies, the overall performance recovery of the full system would be worse than if following a globally-oriented approach (proposed by the INDP models). However, it may be the case that some individual networks may benefit from such individually-oriented approach.

To study this case, we simulated 100 different disaster realizations (consistent with the damage scenarios described in Section 4.1), for which we used the td-INDP to find the recovery strategy that minimizes the global recovery and operation costs. For this case, we assume that the system can repair as many as three elements per period. Additionally, for the same disaster scenarios, we also found the recovery strategies that minimize the recovery and operation costs for each individual infrastructure network, assuming that each network does not take into account their interdependencies with the others. This can be done by using the td-INDP as well, by simply solving the single-network scenario for each network separately. For this case, we assume that each network can repair only one element per period (such that the whole system can recover up to three elements per period).

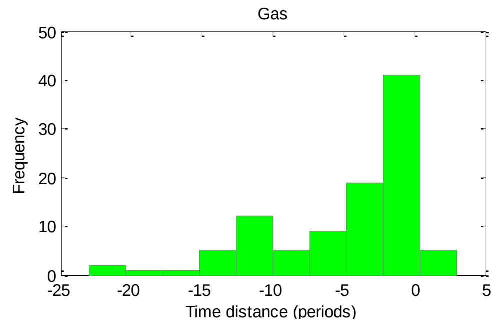
Now, determine the times in which each component regains functionality, related to following either the globally-oriented or the individually-oriented recovery strategy. For the globally-oriented strategies, determining the times in which each component regains functionality is straightforward, as this is indicated directly by the variables y_{ijkt} and w_{ikt}. However, in order to determine these times for the individually-oriented strategies, it would be necessary to assume that each network follows their own reconstruction plan, but the final element functionality states would depend on the other networks' functionality (as these networks are actually interdependent). Then, for the individually-oriented case, it can be seen that the real time in which each component regains functionality will defer from what was originally expected when designing the reconstruction plan without acknowledging interdependencies.

Depending on the optimization strategy used (globally- or individually-oriented), each component of each network will be recovered at a different time. If, for each studied disaster scenario, we add up the differences in functionality-recovery times over all components, we can estimate which strategy results in an overall faster recovery for each infrastructure. Figure 5.1 shows a histogram of the aggregated functionality-recovery time differences between the globally- and individually-oriented strategies, related to each of the studied disaster realizations. A positive time difference (or time distance) would indicate that the aggregate functionality-recovery time for the globally-oriented strategy was greater than the corresponding individually-oriented strategy, and a negative time difference would indicate the opposite.

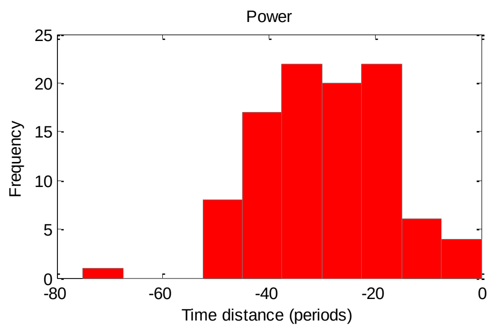
Figure 5.1a shows that for all disaster realizations, the globally-oriented strategy had an aggregate faster recovery than the individually-oriented strategies. Again,



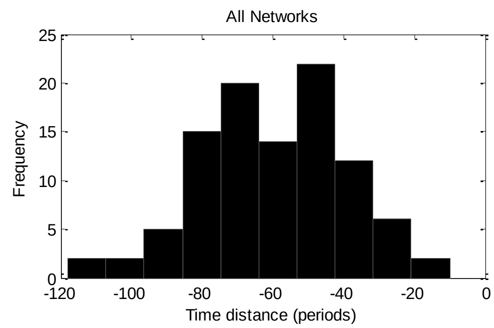
(a) Water



(b) Gas



(c) Power



(d) Global

Figure 5.1 : Histogram for the aggregate differences between element-wise recovery times, associated with globally-oriented and individually-oriented recovery strategies

this effect is seen because, if the networks do not share information regarding their damage state, the plan designed specifically to recover the water network will suffer from delays due to its dependencies to other networks. Similarly, Figures 5.1b and 5.1c show the same negative impact just described. However, note that for a few cases, the individually-oriented strategy was indeed better for the gas network, which may indicate that the recovery of the gas networks may be more independent than the other networks. Finally, 5.1 shows that for all studied scenarios, the recovery strategies designed for the full system performed better than the recovery strategies resulting of each individual network designing for its own benefit only, and without sharing information.

INDP & game theory

Recent works, such as Sharkey et al. (2015), acknowledge the importance of sharing information between infrastructure systems, particularly when modeling the restoration of interdependent systems. Nevertheless, they often assume that the information shared by each network always reflects detailed and accurate recovery plans, without taking into account that these plans may not take place in the exact proposed times and places. In real infrastructure systems, rather than sharing such detailed recovery plans, the networks would probably share estimates of the recovery times for each component (or even per service areas), making it important to account for such an uncertainty and imperfect information.

Additionally, such models assume that for each network the main incentive to share their recovery plans is based on obtaining information that would improve their own recoveries; however, additional types of incentives and dynamics should be considered for realistic purposes, such as particular utility functions or non-cooperative considerations.

In order to address these decentralization challenges, there are diverse approaches that could be implemented, based on combining diverse decision theory approaches and the proposed td-INDP. For example, one could use agent-based modeling (Casalicchio et al., 2010; Permann, 2007; Nejat and Damnjanovic, 2012; Basak et al., 2011; Oliva et al., 2010) or game-theory approaches, among others (Zhang et al., 2005; Yaïche et al., 2000; von Neumann and Morgenstern, 2007).

In particular, Reilly et al. (2015) proposed a general game-theory framework to model interdependent infrastructure systems. Using their model, they could analytically study the differences between a decision maker with globally-oriented goals and one with individually-oriented goals, providing useful tools to develop public policies and incentives for improved resilience. However, their approach mostly focuses on modeling and understanding strategic behaviors from each system, but it does not model the actual operation and recovery of the systems.

Improving upon the models proposed by Sharkey et al. (2015) and Reilly et al. (2015), we proposed an integrated game theory & INDP modeling approach (Smith et al., 2016). This modeling approach, denominated the Interdependent Network Recovery Games (INRG), enables modeling the operation and recovery process of a system of interdependent networks, while also considering interactions between different stakeholders. In particular, we assumed that each infrastructure network was managed by a different decision maker. Each of these decision makers were modeled as 'players' that interacted under a game-theory framework. To model the exchange of imperfect information, we assumed that each player only had information about the elements of its own network, as well as about the elements from other networks in which they depend on (as they can observe them directly). Additionally, to model the objective function of each player, we considered the cost structure proposed in the td-INDP objective function, but calculated it only for each particular network individually.

Figure 5.2 shows a comparison between the total recovery costs associated with an iINDP approach using $t_{horizon} = 1$ (labeled 'td-INDP,$\|T\|$= 1'), an iINDP approach using $t_{horizon} = 5$ (labeled 'td-INDP,$\|T\|$= 5'), the best-case-scenario information-sharing model proposed by Sharkey et al. (2015) (labeled 'InfoShare (Sum)'), and our proposed game theory approach (labeled 'Rand. INRG-BR'). The modeled recovery process corresponds to the interdependent system of water and power networks in Shelby County, TN. As expected, the Figure shows that the total cost associated with the globally-oriented recovery were the lowest, while the INRG approach was the highest. However, note that both decentralized optimization approaches had resultant recovery costs of similar order, despite of one (the INRG) only considering imperfect information.

In order to gain additional insights regarding the impact of considering a decentralized approach, Figure 5.3 shows an estimation of the Price of Anarchy (PoA) –calculated as the ratio between the average cost of recovery in a decentralized approach and the average cost of recovery in a centralized approach– using the information-sharing model proposed by Sharkey et al. (2015) (labeled 'InfoShare'), the proposed game theory approach (INRG) with the players taking decisions in a random order (labeled 'Rand. INRG-BR'), the INRG approach with the power network taking decisions only after the water network (labeled 'INRG-BR (Water lead)'), and the INRG approach with the water network taking decisions only after the power network (labeled 'INRG-BR (Power lead)'). This Figure shows that, independently of the decentralized model used, the PoA increases with the magnitude of the earthquake. This is expected, since the larger the earthquake, the more elements will tend to be damaged, resulting in a larger number of possible recovery strategies for each player. However, note that the price of anarchy calculated using the information-sharing model increases much slower that the one calculated using the INRG approaches. This is consistent with the fact that the information-sharing model assumes fully cooperative individuals and perfect information, in contrast with the INRG where the players may be non-cooperative, and where the information available to each player is limited. Additionally, note that Figure 5.3 shows that the solving times associated with the information-sharing model far surpass the ones associated with the INRG approaches, which is expected considering that the information-sharing model would only converge when it finds solutions that satisfy all players simultaneously, which is not always achievable (Smith et al., 2016).

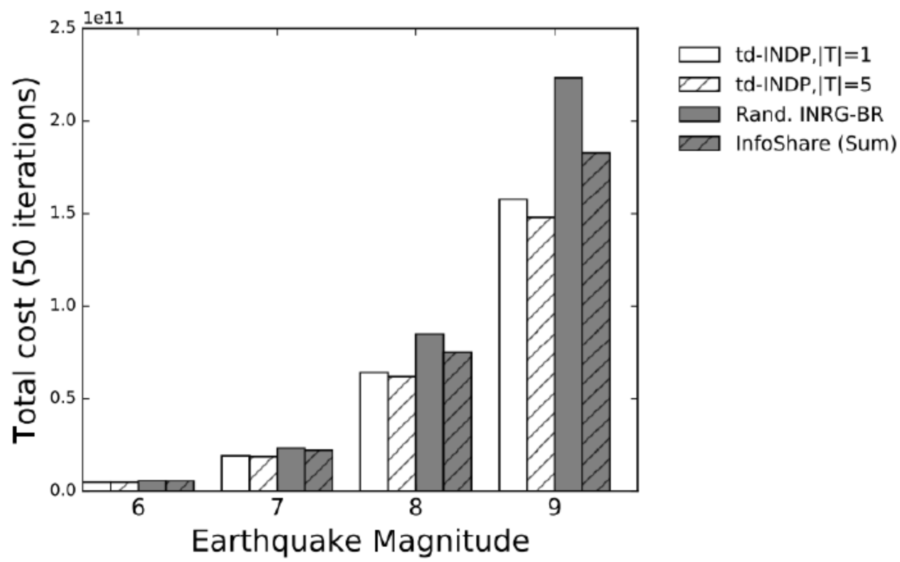


Figure 5.2 : Comparison between the solutions offered by the information-sharing model, the iINDP method, and the INRG (Smith et al., 2016).

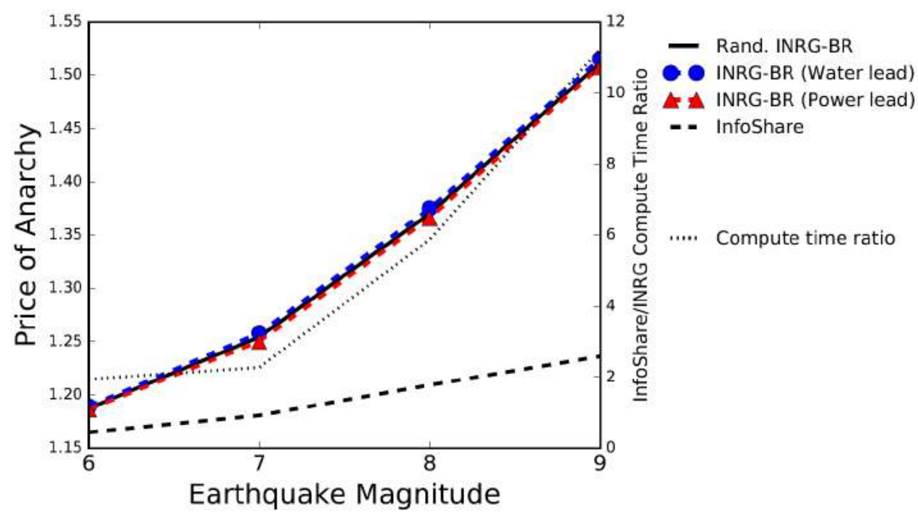


Figure 5.3 : Price of anarchy computed using the information-sharing model and the INRG framework (Smith et al., 2016).

5.2 System identification techniques applied to infrastructure recovery

[†]Providing good recovery strategies for a given networked system is in general a complex task. In particular, using time-dependent mixed-integer programming (MIP) models (such as the models proposed by Lee II et al. (2007); Cavdaroglu et al. (2011); Nurre et al. (2012), or the one presented in Section 2.2) may be computationally expensive, and for scenarios in which it is necessary to obtain results in real time, they may not be suitable. One option to overcome such a problem is to pre-calculate prototypical disaster configurations and their respective recovery strategies, to identify and assess the associated risks, vulnerabilities, and challenges, and to develop pre-disaster recovery plans (U.S. Federal Emergency Management Agency (FEMA), 2011). Unfortunately, given the large number of feasible disaster configurations, creating and maintaining plans for a catalogue of possible disasters becomes a prohibitive task. Thus, an alternative option described in this Section is to create a system identification (system ID) framework that would take families of prototypical failure scenarios and their respective optimal recovery strategies as the input/output data to be studied, which in turn can be used to generate tailor-made recovery strategies. In particular, we propose constructing a linear operator, following a truncated Koopman operator approach –which has been proved to be able to approximate non-linear dynamics– (Brunton et al., 2016; Koopman and von Neumann, 1932), to approximate the recovery dynamics observed in a system of interconnected networks. The proposed system ID framework uses td-INDP input/output 'snap-shot' data to divulge key features of the system, its approximate recovery dynamics, and provide tools to analyze and generate

[†]This Section is based on the submitted pre-print of González et al. (2017a). The definitive version of this paper will be available at www.onlinelibrary.wiley.com and www.blackwell-synergy.com

good recovery strategies in an efficient manner.

5.2.1 Time-invariant recovery operator $\hat{A}$

Assume there is a set of elements E for which we want to study and model their failure and recovery dynamics. Without loss of generality, since the system of infrastructure networks is being modeled using graphs, assume that the set of elements may refer to the nodes in the system (i.e., $E = \mathcal{N}$), the arcs (i.e., $E = \mathcal{A}$), or both (i.e., $E = \mathcal{N} \cup \mathcal{A}$). Now, define $\phi(t)$ as the vector the describes the damage state information for all elements $e \in E$ at time t. Each entry $\phi_e(t)$ of such a vector is either 1 if the element e is damaged at time t, or 0 otherwise.

Let us assume that the recovery process of the system is deterministic, implying that given a disaster scenario the damage state of each element at each subsequent period can be uniquely determined —this is indeed the case when a restoration sequence is found by the td-INDP strategy (but we consider multiple scenarios subsequently). Furthermore, assume that the damage states at a time $t+1$ can be fully determined based only on the damage states at time t —this assumption will be examined in the results section 5.2.2. Then, there is a memory-less mapping f such that:

$$\phi(t+1) = f(\phi(t)) \tag{5.1}$$

In general, this mapping can be described with an operator $A(t)$, such that $\phi(t+1) = A(t)\phi(t)$. What we proposed (González et al., 2017a), is approximating $A(t)$ with a time-invariant operator $\hat{A}$, such that

$$\phi(t+1) \approx \hat{A}\phi(t) \tag{5.2}$$

The operator $\hat{A}$ is selected to minimize the approximation error over all time periods, namely

$$\hat{A} = \arg \min_{A \in \mathbb{R}^{|E| \times |E|}} \|\Phi_0 - A\Phi_1\| \tag{5.3}$$

where (Φ_0, Φ_1) is the input-output history pair

$$\Phi_0 = [\phi(0), \phi(1), ..., \phi(|\mathcal{T}|-1)] \tag{5.4}$$
$$\Phi_1 = [\phi(1), \phi(2), ..., \phi(|\mathcal{T}|)] \,. \tag{5.5}$$

This operator is associated with a given disaster scenario. In order to construct an analogue operator for a set of n disaster scenarios, we can construct $\tilde{\Phi}_0$ and $\tilde{\Phi}_1$ as

$$\tilde{\Phi}_0 = \left[\Phi_0^1, \Phi_0^2, ..., \Phi_0^n\right] \tag{5.6}$$
$$\tilde{\Phi}_1 = \left[\Phi_1^1, \Phi_1^2, ..., \Phi_1^n\right] \tag{5.7}$$

where (Φ_0^i, Φ_1^i) is the input-output pair of disaster scenario i.

Then, it can be seen that in this case

$$\hat{A} = \arg \min_{A \in \mathbb{R}^{|E| \times |E|}} \|\tilde{\Phi}_1 - A\tilde{\Phi}_0\| \tag{5.8}$$

If we assume that the norm used to calculate the error term in Equation (5.8) is the Frobenius norm (the square root of the sum of the squares of each entry), then the optimization problem associated with finding $\hat{A}$ is highly efficient, and the minimizer of this problem assumes the closed form

$$\hat{A} = \tilde{\Phi}_1\tilde{\Phi}_0^T(\tilde{\Phi}_0\tilde{\Phi}_0^T)^{-1} \tag{5.9}$$

where $\tilde{\Phi}_1\tilde{\Phi}_0^T$ denotes the one-period temporal cross-correlation, and $(\tilde{\Phi}_0\tilde{\Phi}_0^T)^{-1}$ is associated with the auto-correlation of the damage states. The operator $\hat{A}$ can be viewed as a matrix of dimensions $|E|\times|E|$ composed of real values. In general, each position $\hat{A}_{\hat{i}\hat{j}}$ indicates the influence that the damage state of element $\hat{i}$ at any given time t has over the damage state of element $\hat{j}$ at time $t+1$. This operator contains important information about the recovery dynamics of the system. In particular, diverse relevant analyses could be performed using the recovery operator, such as finding the recovery modes of the system and their rates of convergence via dynamic mode decomposition, based on the eigendecomposition of $\hat{A}$.

Note that the proposed method to construct the recovery operator assumes that each individual element has the same relative importance, thus errors in their individual recovery times are weighted identically. The method could be expanded to consider prior information about the relative importance of each element or each time period modeled. Assuming there are positive diagonal matrices W_E and $W_{\mathcal{T}}$ that describe the relative importance (weights) associated with the studied elements and recovery periods, respectively, the general optimization problem to construct the recovery operator would then be

$$\hat{A} = \arg \min_{A \in \mathbb{R}^{|E| \times |E|}} \| W_E(\tilde{\Phi}_1 - A\tilde{\Phi}_0) W_{\mathcal{T}} \|. \tag{5.10}$$

Efficient generation of recovery strategies

The simplicity of the linear dynamics (5.2) provides a computationally efficient method to generate approximate recovery strategies given a set of initial damage conditions.

Assume that for a given time t, there is a known damage state $\phi(t)$, and we want to generate a recovery strategy starting from it, namely $\{\breve{\phi}(t), \breve{\phi}(t+1), ..., \breve{\phi}(|\mathcal{T}|)\}$ with $\breve{\phi}(t) = \phi(t)$. Assuming that $\hat{A}$ has already been calculated (which in practice can be done during non-time-critical periods), then we know that for the period $t+1$ the approximate damage state can be calculated by $\hat{A}\breve{\phi}(t)$. As the approximation dynamics is continuous then $\hat{A}\breve{\phi}(t)$ will require a projection step to describe a binary damage state. Thus, we can define a threshold value $0 < \bar{a} < 1$, that will determine the ranges in which a non-integer damage state will be rounded to a binary state. Then, the damage state for period $t+1$ would be

$$\breve{\phi}(t+1) = \lceil (\hat{A}\breve{\phi}(t) - \mathbf{1}\bar{a}) \rceil \tag{5.11}$$

Generalizing for any positive integer m periods in the future, we have that

$$\breve{\phi}(t+m) = \lceil (\hat{A}^m\breve{\phi}(t) - \mathbf{1}\bar{a}) \rceil. \tag{5.12}$$

The proposed method uses the recovery operator to replicate the recovery dynamics from the generating data, without enforcing any constraint on the predicted recovery strategy. If there are constraints that need to be enforced, such as a strict maximum resource utilization per recovery period, it is possible to use the recovery operator to formulate a general iterative approach as follows:

Assume that our current iteration starts from period t and its related damage state is $\breve{\phi}(t)$, then $\hat{A}\breve{\phi}(t)$ can be interpreted as the relative ranking or importance of recovering each node at time $t+1$. The ranking can then be used to determine which set of elements should be recovered at period t, such that all desired constraints are guaranteed. For example, if there is a strict maximum resource utilization, one could select the set of elements with higher importance and an associated total resource utilization less than the maximum allowed for that period. Considering the elements selected to be recovered at time t, calculate $\breve{\phi}(t+1)$, update the vector of relative importances (now $\hat{A}\breve{\phi}(t+1)$), and repeat.

Note that in the case of a recovery process, the spectral radius of the operator (denoted as $\rho(A)$) is expected to be strictly less than one, in order to guarantee that when using any disaster configuration as the initial damage state $\breve{\phi}(0)$, the damage states of the system will converge to a full recovery as t increases (since $\lim_{m\to\infty} A^m = \mathbf{0}$ if and only if $\rho(A) < 1$).

Measuring the quality of the generated recovery strategies

In order to use the strategies that are generated using the recovery operator, it is important to quantify their quality. In general, it is desirable that the recovery strategies generated using the recovery operator can resemble as close as possible strategies considered to be optimal. Using the strategies generated by the td-INDP model as a benchmark, it is possible to evaluate how much the recovery times deviate from one strategy to the other. In particular, the ideal is that the recovery times proposed for each individual component by the recovery operator approach deviate as little as possible compared to the benchmark td-INDP strategy.

In some instances, focusing only on comparing the approximate strategies with the optimal one may not be enough. In particular, even though the recovery times of an approximate strategy may deviate little compared to the optimal one, the resultant performance recovery and costs associated with implementing them may differ greatly. Moreover, the approximate recovery strategy may be an unfeasible one. In particular, if the elements of the studied system are highly interdependent, or if the available resources are very limited, the feasibility of the solutions will be sensitive to changes in the recovery times of each element. Thus, in addition to studying the errors in the estimated recovery times, it is also important to compare the cumulative percentage of repaired elements associated to the estimated and benchmark recovery strategies. If this cumulative function describes different behaviors for the two strategies, this would indicate that the performance recovery, the resource utilization, and the costs associated to each strategy are not consistent.

Whenever there are inconsistencies in the predicted times of recovery or the cumulative percentage of repaired elements, it is important to evaluate the impact of these inconsistencies on the quality of the predicted recovery strategy. In order to estimate the impact of these recovery time deviations in an efficient manner, we can construct an optimization model to both evaluate the performance of a given recovery strategy, and detect any infeasibility associated with it. Such an idea was introduced by González et al. (2016a), where a modified td-INDP formulation was presented with these goals in mind. The modified td-INDP model resembles the td-INDP model shown in Section 2.2, but Δw, Δy, and Δz are no longer used as decision variables, and instead are replaced by analogous parameters defined as ∇w, ∇y, and ∇z which represent the fixed recovery strategy to be studied, where $\{\nabla w, \nabla y\}$ at time t is given by $\breve{\phi}(t+1) - \breve{\phi}(t)$. ∇z is not encoded in $\breve{\phi}(t)$, but it can be easily (and uniquely) determined once ∇w and ∇y are fixed. Then, the performance F of the given recovery strategy $\{\nabla w$, ∇y, and $\nabla z\}$, which measures the cost associated with implementing such an strategy, is given by equation (5.13), and it is subject to the constraints that result after replacing Δw, Δy, and Δz with ∇w, ∇y, and ∇z, respectively, in inequalities (2.1b)-(2.1s),

$$\begin{aligned} F(\nabla w, \nabla y, \nabla z) = \min_{x,\delta^+,\delta^-,w,y} & \sum_{t\in\mathcal{T}|t>0} \left(\sum_{s\in\mathcal{S}} g_{st} \nabla z_{st} \right. \\ & \left. + \sum_{k\in\mathcal{K}} \left(\sum_{(i,j)\in\mathcal{A}'_k} f_{ijkt} \nabla y_{ijkt} + \sum_{i\in\mathcal{N}'_k} q_{ikt} \nabla w_{ikt} \right) \right) \\ & + \sum_{t\in\mathcal{T}} \sum_{k\in\mathcal{K}} \left(\sum_{l\in\mathcal{L}_k} \sum_{i\in\mathcal{N}_k} \left(M^+_{iklt}\delta^+_{iklt} + M^-_{iklt}\delta^-_{iklt} \right) \right. \\ & \left. + \sum_{l\in\mathcal{L}_k} \sum_{(i,j)\in\mathcal{A}_k} c_{ijklt} x_{ijklt} \right). \end{aligned} \tag{5.13}$$

In particular, the studied recovery strategy is unfeasible if the following inequalities do not hold (note that these inequalities are analogous to constraints (2.1k)),

$$\sum_{k\in\mathcal{K}} \left(\sum_{(i,j)\in\mathcal{A}'_k} h_{ijkrt} \nabla y_{ijkt} + \sum_{i\in\mathcal{N}'_k} p_{ikrt} \nabla w_{ikt} \right) \leq v_{rt}, \quad \forall r \in \mathcal{R}, \forall t \in \mathcal{T} \mid t > 0, \tag{5.14}$$

Then, for instances in which the predicted recovery strategies highly deviate from the benchmark strategies, equation (5.13) could be used to evaluate if there was a performance detriment, while inequalities (5.14) would determine if the predicted strategies were not feasible, indicating the exact periods and elements associated with such infeasibility.

5.2.2 Illustrative example

This illustrative example is based on the system of gas, water, and power networks in Shelby County, TN., described in Section 2.3.2, and the disaster realizations described in Section 3.1. For each magnitude $M_w \in \{6, 7, 8, 9\}$, we generated 1000 disaster scenarios, along with their benchmark recovery strategy using the td-INDP model shown in Section 2.2. In order to allow for the number of simulations proposed, we used a truncated td-INDP model, in which if a guaranteed optimal recovery strategy was not found after a fixed running time (120 seconds per simulation), it returned the best found recovery strategy (and its optimality gap). The average optimality gaps between the returned benchmark solutions and their respective best known bounds were 0%, 0%, 1.8%, and 18.8%, for earthquake magnitudes 6, 7, 8 and 9, respectively. These provide analytical guarantees of the quality and accuracy of the benchmark strategies used. To showcase the generality and versatility of the proposed methodologies, the following subsections present two different cases, associated with identifying and approximating the recovery dynamics of the nodes only and of both the nodes and the arcs.

Recovery operator for nodes

Figure 5.4 represents the recovery operator $\hat{A}$ found as proposed in Section 5.2, based on the full set of randomly generated disaster scenarios and their respective td-INDP recovery strategies. Each entry $\hat{A}_{e\tilde{e}}$ (value of $\hat{A}$ at row e and column $\tilde{e}$) shows the strength with which the damage state of element $\tilde{e}$ at any given time period influences the damage state of element e in the next period. The coefficient of determination associated to $\hat{A}$ was $R^2 = 0.7729$, which is a good indicator of the accuracy of the found operator and its predictability power. The spectral radius of this operator is 0.9250, which indicates that the system will tend to full recovery, independently of the initial damage state. Note that Figure 5.4 does not show the lowest values in the operator or its diagonal entries, in order to highlight the non-trivial stronger effects. To select the minimum value to display, first we took all the entries in $\hat{A}$ (except for the diagonal) and sorted their absolute values (in increasing order). Then, we calculated the knee (point of maximum curvature) of the curve associated to this list. Since this point indicates the value for which the rate of growth in the list changes the most, we used this value as the minimum to display. In particular, this value was 0.011 for Figure 5.4. This recovery operator uses $E = \mathcal{N}$, thus it is showing the relation between the damage states of all pair of nodes, with $|\mathcal{N}| = 125$.

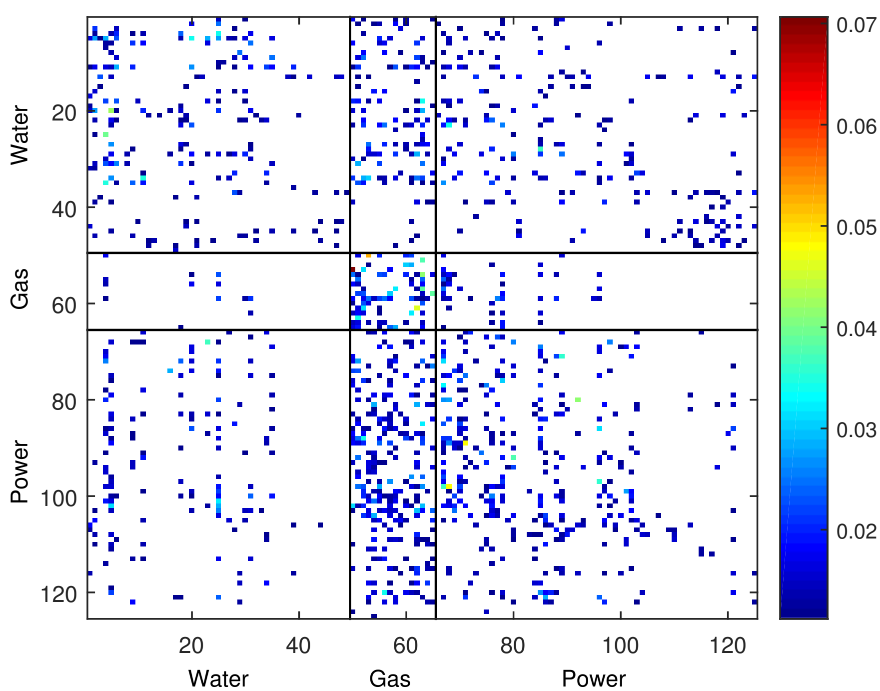


Figure 5.4 : Recovery operator $\hat{A}$ representation (for nodes only) with matrix entries with absolute value below 0.011 not displayed. The rows and columns correspond to the assigned labels for each element (González et al., 2017a).

By drawing horizontal and vertical lines to separate the nodes in each axis into the three infrastructure networks studied, we obtain nine different sections that depict the intra- and inter-network recovery dependencies. For simplicity, let us name each block as $\hat{A}_{k\tilde{k}} : k, \tilde{k} \in \mathcal{K}$, which shows the strength in which the damage state of nodes in network $\tilde{k}$ affect the states of network k. The diagonal blocks (i.e., $\hat{A}_{kk} : k \in \mathcal{K}$) depict the strength in which the current damage state of the nodes in each given network influence the recovery of the nodes of the same network, whereas the off-diagonal blocks (i.e., $\hat{A}_{k\tilde{k}} : k, \tilde{k} \in \mathcal{K} | k \neq \tilde{k}$) depict how the state of the nodes in each network impact the recovery of the other networks. As expected, Figure 5.4 shows that the states of the nodes in each network have a strong influence within the network, but also that some networks have a strong influence on the recovery of the others. In particular, it can be seen that $\hat{A}_{\text{water, gas}}$ and $\hat{A}_{\text{power, gas}}$ are dense compared to the other blocks, showing that the states of the gas nodes have a notable influence over the other networks. This can also be interpreted as a priority to repair the nodes in the gas network before recovering the nodes of other networks —consistent with the td-INDP training data which does not include safety constraints. Such trends can be explained considering the relative topologies of each network, since the studied gas network has fewer elements and much less redundancies than the other networks, which implies that initial repairs in the gas networks have a larger influence on the overall performance of the system of networks (assuming that M^{+}_{iklt} and M^{+}_{iklt} are equal for all networks). On the other side, blocks $\hat{A}_{\text{gas, water}}$ and $\hat{A}_{\text{gas, power}}$ are sparse compared to the others, which indicates that states of the gas network do not strongly depend on the states of the others. This also supports the interpretation that the gas network had a priority to be recovered. Note that these influence relationships are not trivially observable without relying on the recovery operator, since as mentioned, the gas was not considered to be physically dependent on the power or the water in the training td-INDP data. Note that the element ordering used in Figure 5.4 was adequate to easily separate the elements from different networks, but other element sorting may also be useful. For example, one could sort the elements such that their closeness in the operator reflects its geographical co-location, providing a graphical depiction that would facilitate finding the influence that each geographical region has over others.

Generated recovery strategies for nodes

In order to evaluate the power and accuracy of the proposed methodology to efficiently generate recovery strategies using the recovery operator, we separated all disaster scenarios into ten different groups (of 100 scenarios each) and applied cross-validation. In particular, for each group we generated recovery strategies, using a recovery operator created using only the benchmark (td-INDP-generated) recovery strategies of the other nine groups. For each disaster scenario in each group, seeded with the initial disaster state $\phi(0) = \breve{\phi}(0)$, we used its associated recovery operator to efficiently generate a recovery strategy as described in Section 5.2.1. The generated strategies were compared with their associated td-INDP-based recovery strategies, to study their quality of approximation for each time period. For this case study, assuming no prior information was provided, we used a threshold value –number used in equation (5.12) to round a non-integer damage state to a binary state– of $\bar{a} = 0.5$. Figure 5.5a shows the box-plot of the errors between the predicted strategies with respect to their benchmark, the number of elements with repair times accurately predicted for each period, and the cumulative percentage of recovery for both the predicted and the benchmark strategies. Figure 5.5b shows the associated histogram of the errors between the two strategies. It can be observed that the predicted recovery strategies can replicate the td-INDP recovery strategies very accurately, given that the median error is less than one period, and in general the estimated recovery times have an error below three periods. In fact, the overall distribution of errors shows that more than 50% of the errors are within only one period from the median, and more than 75% within two periods of the median, indicating that the generated strategies are also precise. Note that even though the accuracy is better for the starting recovery periods, a good level of accuracy is achieved even when performing long range prediction (i.e., when estimating recovery times that are far from the current time).

Note that the quality of the estimated solutions depends directly on the threshold value $\bar{a}$ used. For higher accuracy, parameter $\bar{a}$ could be tuned in order to minimize the difference between the benchmark and the predicted recovery, i.e., by solving an additional optimization problem of the form $\arg\min_{\bar{a}} \|\breve{\phi}(t+k) - (\hat{A}^k \breve{\phi}(t) - \mathbf{1}\bar{a})\|$. To exemplify the effects of choosing a 'tuned-up' $\bar{a}$, Figures 5.6a and 5.6b show a comparison between the benchmark and the predicted recovery times using $\bar{a} = 0.4$. From Figure 5.6a it can be seen that the relative number of outliers observed for each recovery time is less than the one observed using $\bar{a} = 0.5$. Similarly, the predicted cumulative fraction of elements recovered is much more accurate than if using $\bar{a} = 0.5$. In particular, it is important to ensure that the predicted cumulative fraction of elements recovered is not extensively far above the benchmark, as it would be suggesting likely unfeasible recovery strategies that are using more resources than available. Moreover, Figure 5.6b shows that the histogram of the observed errors is better centered around zero, relative to its analogous in Figure 5.5b.

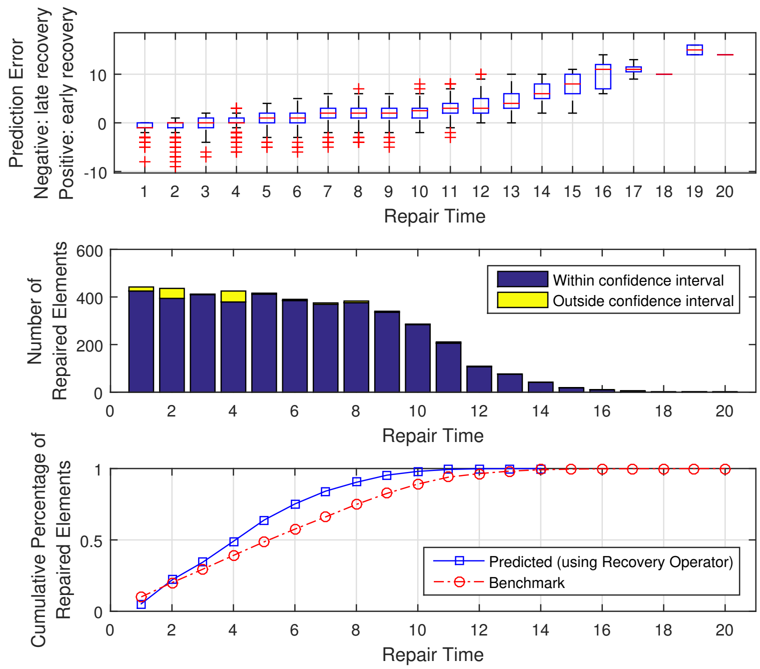
To provide some insight on how equation (5.12) is used to generate the recovery strategies, Figure 5.7 shows the evolution of the failure states for a set of 10 nodes, comparing their td-INDP recovery strategy and their associated values of $\hat{A}^k \breve{\phi}(0)$. For each component, a value of 1 describes a fully failed, whereas 0 describes a fully recovered state. For each element in Figure 5.7, there is a gray box that highlights in the horizontal axis the time period in which the element was recovered according to the td-INDP strategy, and in the vertical axis the range of values that $\bar{a}$ can take such that the generated recovery strategy perfectly replicates the td-INDP strategy. As expected, the faster an element is recovered according to the td-INDP strategy, the larger is the range of values that $\bar{a}$ can take to return an accurate recovery strategy based on the recovery operator, since the slope of $\hat{A}^k\breve{\phi}(0)$ for that particular element would be higher. For example, it can be observed that the range of threshold values that would generate an accurate recovery time for element 36 is much narrower than the same range associated with element 5. Note that another approach to fine-tune $\bar{a}$ would be finding the threshold value that better fits these accuracy ranges across all elements and scenarios. In this case, $\bar{a}$ would maximize the number of times in which the recovery periods were accurately predicted, instead of minimizing the overall error.

Recovery operator for nodes and arcs

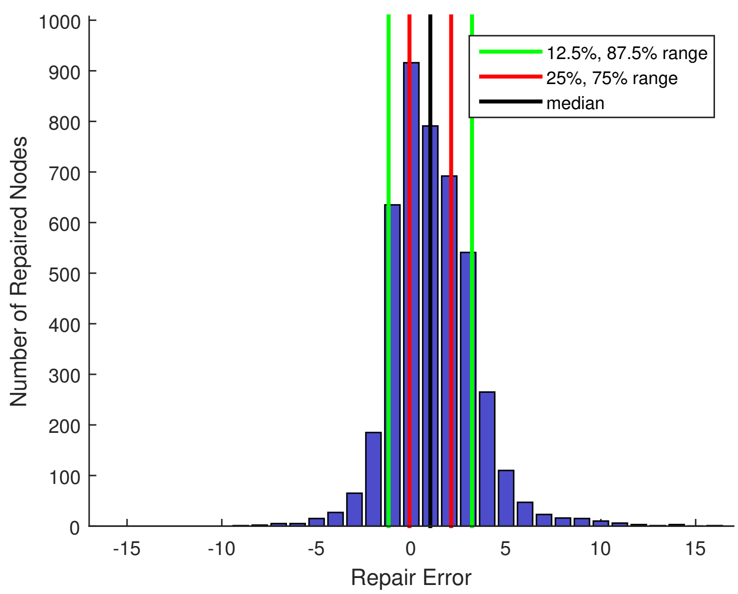
Now, in order to model the recovery process of all the elements in the studied system of networks, assume $E = \mathcal{N} \cup \mathcal{A}$. In this case, the total number of elements increases from 125 to 289. Figure 5.8 shows the calculated recovery operator $\hat{A}$ for this new set of elements, using a display threshold of 0.0079. The coefficient of determination associated with this operator was $R^2 = 0.8857$, which shows an increase in its predictability power due to the inclusion of additional significant elements. The spectral radius of this operator is 0.9944, which again provides theoretical guarantees of a predicted full recovery of the system. As observed in the recovery operator associated with nodes only, the blocks $\hat{A}_{\text{water, gas}}$ and $\hat{A}_{\text{power, gas}}$ are dense relative to other blocks, whereas $\hat{A}_{\text{gas, water}}$ and $\hat{A}_{\text{gas, power}}$ are the most sparse. This shows that the recovery operator provides consistent results regarding the relationships between networks. Nevertheless, it can be seen that for each block in this operator, the upper-left corners (associated with the node elements, as opposed to the arc elements) are densest. This indicates that in general the recovery relationships between nodes tend to be stronger than the ones observed for the arcs.

Generated recovery strategies for both nodes and arcs

Following the procedure detailed in Section 5.2.2, for each of the disaster scenarios, we generated recovery strategies for both nodes and arcs simultaneously. For this case, the threshold value $\bar{a}$ used was also 0.5. As in the previous examples, Figure 5.9 overviews different aspects associated with the errors of the generated recovery strategies compared to the td-INDP strategies. Figures 5.9a and 5.9b show that the generated strategies are also accurate, having a median error equal to 0, but with a slightly increased variability. In particular, Figure 5.9 shows that the error distribution has a reduced symmetry compared to the one associated with nodes only, favoring outliers in the left tail. This may be explained by the weaker recovery relationships observed in the arcs compared to the ones observed between the nodes, reducing the predictability power for these types of elements. Nevertheless, despite this increased variability, more than 50% of the proposed recovery times are within two periods with respect to the td-INDP strategies, and more than 75% within 5 periods.

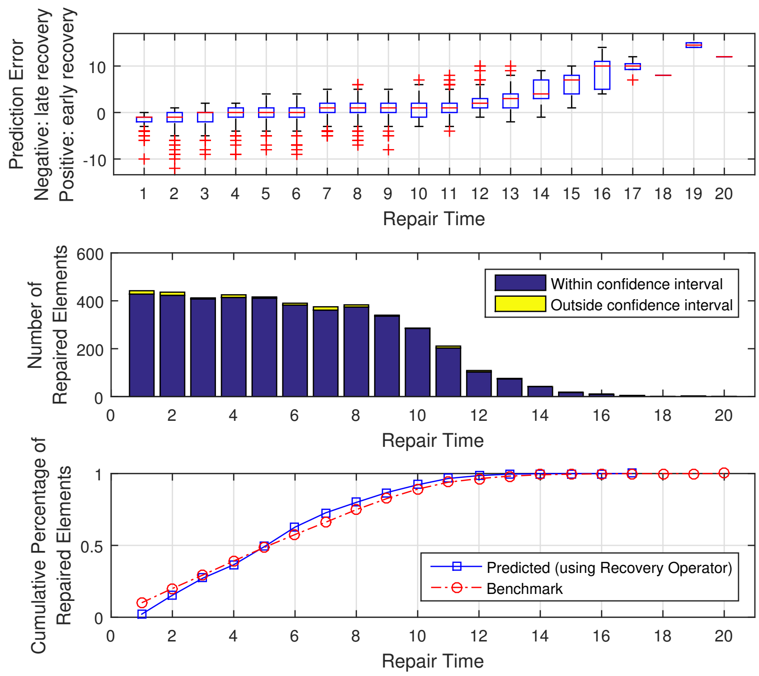


(a) Box-plot of the errors, number of repaired elements, and cumulative percentage of recovery, respectively, for each recovery period.

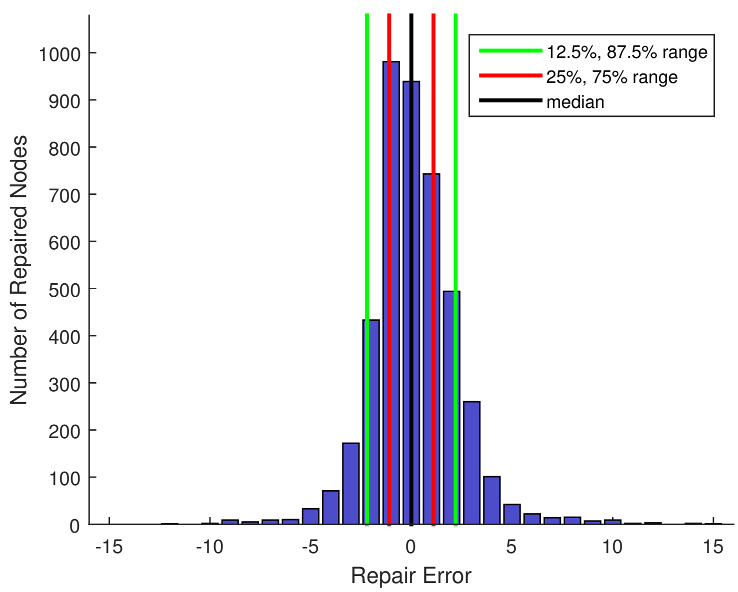


(b) Histogram of the errors

Figure 5.5 : Comparison between predicted recovery strategy for nodes (using the recovery operator) and the benchmark strategies (based on the td-INDP), using $\bar{a} = 0.5$ (González et al., 2017a).



(a) Box-plot of the errors, number of repaired elements, and cumulative percentage of recovery, respectively, for each recovery period.



(b) Histogram of the errors

Figure 5.6 : Comparison between predicted recovery strategy for nodes (from the recovery operator) and the benchmark strategies (based on the td-INDP), using $\bar{a} = 0.4$ (González et al., 2017a).

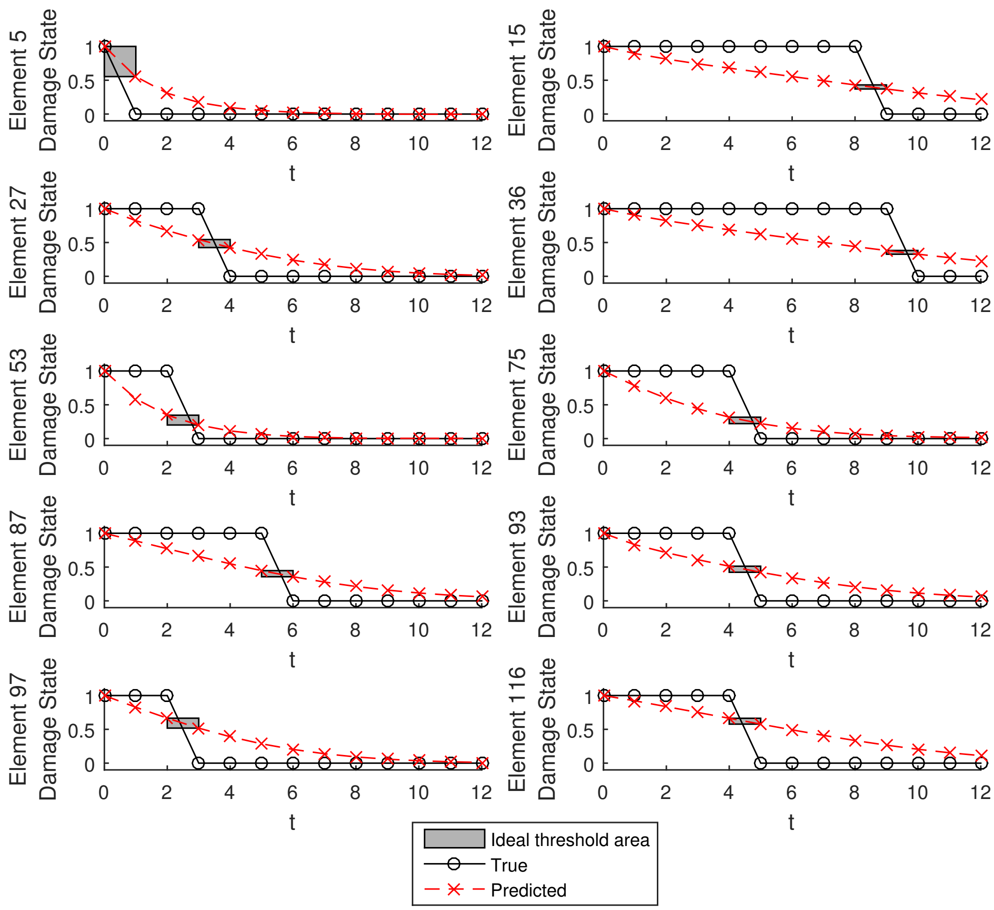


Figure 5.7 : Recovery state estimation for 10 randomly selected components (González et al., 2017a)

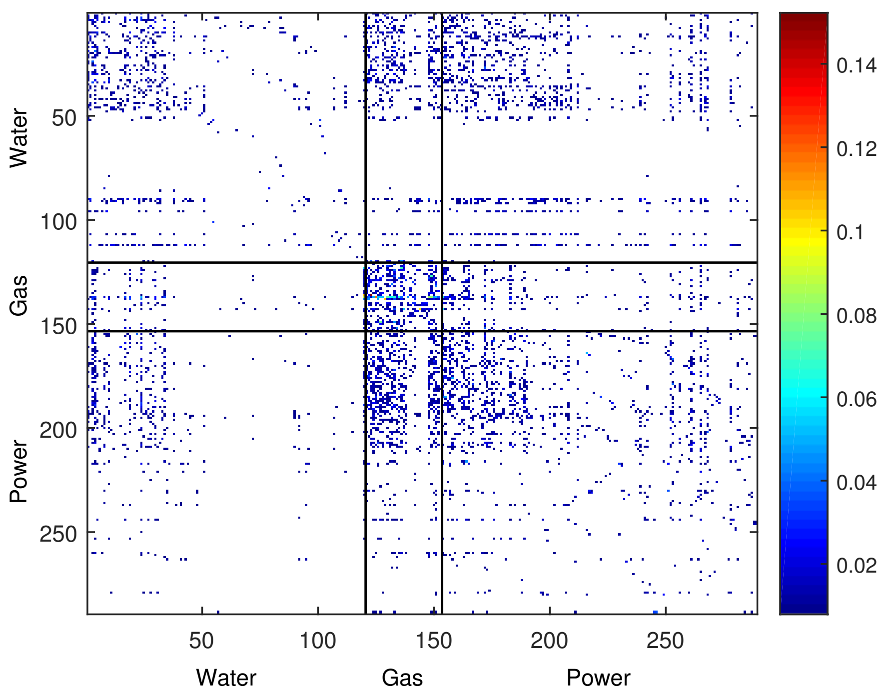
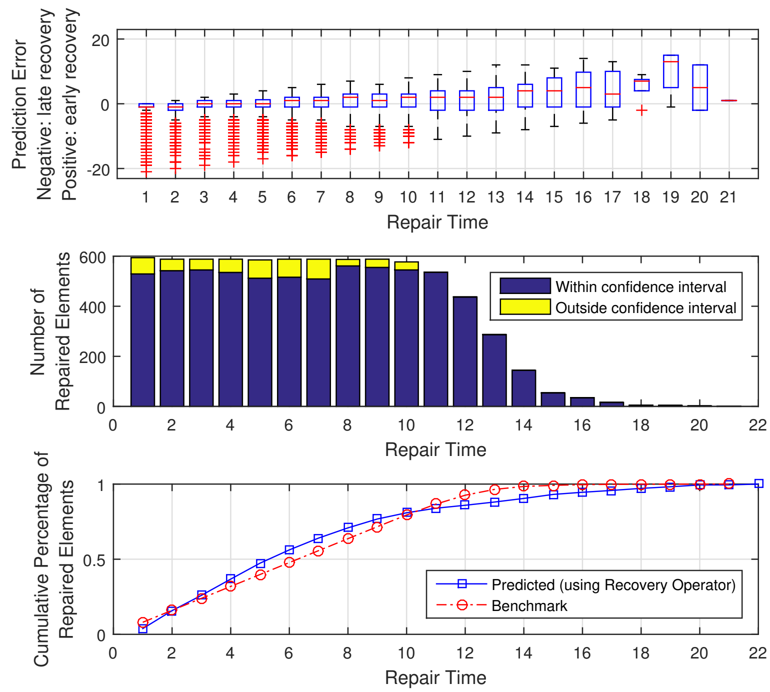
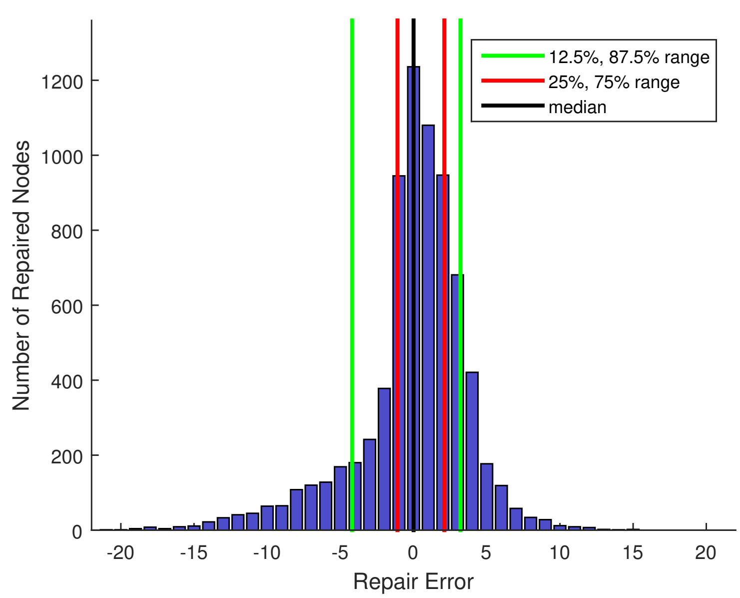


Figure 5.8 : Recovery operator $\hat{A}$ representation (including both nodes and arcs) with matrix entries with absolute value below 0.0079 not displayed. The rows and columns correspond to the assigned labels for each element (González et al., 2017a).



(a) Box-plot of the errors, number of repaired elements, and cumulative percentage of recovery, respectively, for each recovery period.



(b) Histogram of the errors

Figure 5.9 : Comparison between predicted recovery strategy for nodes and arcs (from the recovery operator) and the benchmark strategies (based on the td-INDP), using $\bar{a} = 0.5$ (González et al., 2017a).

5.3 Conclusions

This Chapter presents a set of multidisciplinary extensions for the td-INDP, focused on expanding the model capabilities for enhanced realistic scenario analysis.

Section 5.1 shows the impact of decentralized optimization, specifically demonstrating the effects of optimizing under imperfect information. Using an illustrative example, based on the system of gas, water, and power networks from Shelby County, TN (described in 2.3.2), we showed that following recovery strategies constructed without considering the effects of interdependencies lead to important delays in the recovery process. We also discussed the existing limitations related to modeling multiple interacting decision makers, and proposed an extension of the td-INDP that embeds it in a game-theory framework, denominated the Interdependent Network Recovery Games (INRG). We show that by using such td-INDP game-theory framework, it is possible to accurately model the recovery and operation of a system of interdependent networks, while considering multiple agents in charge of the recovery of each network in the system. The presented game-theory approach permits modeling multiple realistic characteristics of realistic systems, such as having multiple possibly non-cooperative agents in charge of the recovery process for different parts of the system. Also, we showed that the proposed INRG framework can be used to estimate relevant metrics associated with systems managed by multiple interacting decision makers. In particular, we showed that the INRG framework can be used to estimate the Price of Anarchy (PoA) of a system of interdependent networks, such as the system of power, water, and gas networks in Shelby County, TN.

As with any general game-theory model, the proposed INRG framework is based on the assumptions that each player acts rationally and intelligently, following a well-defined utility function, and with full knowledge of the recovery strategies available to him and the other players at each recovery period. However, in certain circumstances such assumptions may not be realistic, since human behavior often does not follow a course of action defined by single well-known rational utility functions. In order to address such concerns, one could use other approaches such as agent-based modeling, which facilitate modeling irrational behavior and individual learning processes, among others.

Section 5.2 presented a novel approach to efficiently generate approximate recovery strategies for interdependent infrastructure systems, using a linear memory-less recovery operator. The recovery operator is generated from a set of high-fidelity recovery scenarios, in order to identify and approximate their main recovery dynamics, and then to develop tailor-made recovery strategies after specific disaster scenarios. The shown illustrative example is also based on the system of utility networks in Shelby County, TN., for which we used the td-INDP to generate the used benchmark recovery strategies associated with the disaster realizations described in Section 4.1. The recovery strategies generated using the proposed recovery operator approach show good agreement with the td-INDP recovery dynamics, even for long-term planning, both in the times of recovery and the resources used. The proposed recovery operator is expected to adequately approximate the recovery dynamics of a system of interdependent networks, as long as these dynamics are consistent along the full planning horizon. Nevertheless, if parameters such as costs, resources, or capacities change abruptly in time, a single memory-less time-invariant operator may not be appropriate to approximate the system dynamics. However, in such a case, it would be possible to construct recovery operators specifically designed for different recovery periods. In such a case, the full planning horizon would be divided into multiple separate time-domains, and for each of these one could construct a separate recovery operator. This would enable, for example, modeling recovery processes with multiple inflections points (which is difficult to model with other approaches, such as the iINDP). Note that the proposed recovery operator approach is a general data-driven process that operates on 'snap-shots' of input/output data; thus, it is not restricted to any particular optimal recovery model.

Chapter 6

Conclusions and Future Work

In this thesis we have discussed the importance of understanding, modeling, and optimizing the resilience of systems of interdependent networks, such as critical infrastructures. To this end, we presented multiple concepts and methods of analysis, in line with three main observed challenges: modeling and optimizing interdependent networks; facilitating the study of large problems; and, enabling the consideration of multiple sources of uncertainty. Below is an overview of the main topics and methods presented, along with their main conclusions.

Chapter 2: On properly defining the Interdependent Network Design Problem (INDP), and developing computational models to solve it.

- We described the main characteristics that must be considered by a problem related to maximizing the resilience of a system of interdependent networks. This problem was defined as the Interdependent Network Design Problem (INDP).
- We defined the INDP as the problem of finding the least-cost recovery strategy of a partially destroyed system of infrastructure networks, subject to budget, resources, and operational constraints, while considering interdependencies between the studied networks.
- We proposed an MIP formulation to solve the INDP, denominated the time-dependent INDP (td-INDP), which can be used to model and optimize the

recovery of interdependent systems, while considering realistic operational constraints. We showcased the td-INDP using two different illustrative examples, which are subject to an earthquake hazard: the Colombian road network and its multicommodity cargo flow; and, the system of interdependent infrastructure networks composed by the gas, water, and power networks in Shelby County, TN.

The proposed td-INDP is, to the best of our knowledge, the first optimization model that can determine the minimum-cost recovery strategy associated with a system of interdependent networks, which considers realistic operational, resource availability, and capacity constraints, while simultaneously accounting for multiple types of interdependencies, such as physical and geographical. In particular, the proposed physical-interdependency modeling enables determining critical recovery jobs that result in a cascading-recovery effect. Additionally, the geographical-interdependency modeling facilitates cost reductions due to simultaneous recovery of co-located components with shared physical spaces.

Furthermore, the td-INDP model can also be used to provide benchmark recovery strategies that can be used in other academic and practical studies. Considering this, as part of our research, we developed a benchmark failure/recovery time-series dataset associated with the system of water, gas, and power networks in Shelby County, TN. (Appendix C). Taking into account that the proposed optimization model provides optimality guarantees on the obtained solutions, this dataset has been used as a reference by multiple research groups, for studies in diverse fields such as statistical physics and system identification, among others.

Chapter 3: On exploring enhancement techniques for the computational capabilities of the INDP models, such that they run faster and can handle larger systems.

- We developed the iterative INDP (iINDP), which is a heuristic approach that relies on decomposing the full time horizon of a given problem into smaller more manageable time horizons.

- We showed that the time complexity of the td-INDP model is highly dependent on the size of the system analyzed, its level of damage, and availability of resources, but not as much on its link density, or its interdependency density and strength.

- We described the special structure associated with the td-INDP, which makes it suitable for diverse decomposition techniques, such as Dantzig-Wolfe and Benders decompositions.

The proposed td-INDP was carefully developed such that it not only features the aforementioned modeling capabilities associated with realistic interdependent systems, but also sustains a desirable block structure. This structure not only enables solving the td-INDP more efficiently, but also facilitates building efficient modeling extensions, which include stochastic and decentralized formulations.

Additionally, the features depicted by the proposed heuristic and analytical approaches may be combined in tailor-made hybrid methodologies, which can exploit the advantages of each individual approach. For example, heuristic algorithms may be used specifically to determine efficient reconstruction strategies, while analytical methodologies focus on quantifying and optimizing the associated flow and supply of commodities.

Chapter 4: On expanding the INDP models to allow the inclusion of multiple sources of uncertainty.

- We emphasized two sources of uncertainty relevant in the measurement of resilience in a system: uncertainty about the hazard-induced failure modes of the system; and, uncertainty about the system's properties, such as costs, demands, resource availability, and resource utilization, among others.

- We described how to account for uncertainty related to the possible damage of components in a system of interdependent networks. Particularly, we presented a Monte-Carlo simulation framework to enable pre-event analyses on the resilience of systems, to help planning for budget and resource allocation, among others.

- We proposed three different element-wise metrics of relevance for resilience: failure-likelihood; recovery-likelihood; and, recovery-time metrics. We show that these metrics represent importance rankings that are consistent, independently of the analyzed disaster magnitude.

- We introduced a stochastic mathematical formulation, denominated the stochastic INDP (sINDP). This formulation enables determining the recovery strategy that optimized the expected performance of the system, while considering uncertainty in the system's properties, such as demands and resources.

- We showed that the proposed sINDP is susceptible to decomposition techniques, such as the L-shaped method, since it can be formulated as a two-stage stochastic program.

In contrast to pure simulation-based approaches, stochastic formulations, such as the proposed sINDP, provide analytical optimality guarantees on their solutions. Additionally, such formulations facilitate the construction of theoretical lower and upper bounds, which can be later used to study the quality of other modeling and optimization approaches.

Moreover, by extending the capabilities of the proposed INDP models, so that they can consider the aforesaid sources of uncertainty, the upgraded modeling approaches provide efficient tools to support pre- and post-event decisions, which enhance and facilitate mitigation and emergency response actions. Thus, used in combination, these models enable examining and quantifying the effects of interdependencies in realistic interdependent infrastructure systems, as well as optimizing their overall resilience. Under such a context, the proposed models could be used to study the trade-offs between pre-event mitigation processes and the post-event recovery actions, offering important information to policy makers and stakeholders. For example, assuming that the total budget invested on infrastructure retrofitting and recovery cannot surpass a specific threshold level, one could use the proposed models to determine which type of interventions would have a larger impact on the overall resilience of the system of infrastructure networks studied.

Chapter 5: On studying diverse extensions and applications of the proposed INDP computational models, either for practical or theoretical purposes.

- We described the importance of including socio-technical constraints in our models, such that they better depict realistic systems, particularly regarding the interaction between different decision makers in charge of different networks.

- We presented a system identification technique that allows extracting the main recovery dynamics for any given system. In particular, we describe how the recovery dynamics can be compressed in a time-invariant linear operator, denominated the recovery operator. This operator can be used to provide insightful information regarding the recovery relations between elements, as well as to construct efficient recovery strategies in polynomial time.

The presented extensions and applications show that the proposed INDP models are versatile, and can be used in combination with diverse approaches and techniques from other fields and disciplines.

For example, methodologies such as the Interdependent Network Recovery Games (INRG) show that game-theory and the proposed INDP models can be combined in order to optimize the resilience of systems of interdependent networks, while modeling realistic socio-technical features associated with decentralized decision making.

Moreover, the proposed recovery operator not only offers tools to understand and recreate the dynamics related to the recovery of interdependent systems, but also facilitates the development of future models and metrics for the observability and controllability of such systems.

Figure 6.1 presents a conceptual map that summarizes the main areas and concepts covered in this thesis, including some relevant topics for future research.

Opportunities for future research

Even though this thesis presents a comprehensive study on multiple computational and modeling techniques designed to study and optimize the resilience of interdependent networks, this field offers multiple research opportunities for future work.

On one hand, we believe it is important to develop adequate mechanisms to facilitate the proliferation of optimization-based analytical and computational techniques (such as the ones proposed in this thesis) in the field of Civil and Infrastructure Engineering. For example, we could apply the proposed optimization techniques to study and optimize the recovery dynamics of well known systems of infrastructure networks. In line with this idea, as part of this research we developed a hazard-to-recovery benchmark database associated with the system of water, gas, and power networks in Shelby County, TN. (described in Appendix C), such that other researchers can use the included disaster realizations and recovery strategies to calibrate their own models. Similar databases could be developed for other well-known systems, such as the infrastructure networks associated with the virtual city Centerville (Ellingwood et al., 2016). Using these known systems to perform comparisons between our proposed models and others commonly used in Civil and Infrastructure Engineering (such as physics-based and quadratic flow-based models) would facilitate uncovering advantages and disadvantages associated with each approach, which ultimately will result in the development of improved methodologies, positively impacting the field from theoretical and a practical perspectives. Additionally, it is important to ensure that the proposed analytical and computational models address the needs of real infrastructure operators and stakeholders. It is important to always keep in mind that optimality in solutions to a mathematical problem may differ from ideal real-life decisions, if the proposed mathematical models do not accurately represent the operation and the decision making processes of the real systems. To this end, it is important to actively participate in programs and initiatives oriented towards facilitating a close communication with policy makers, private and public infrastructure operators, and final users and consumers –such as the ones proposed by the infrastructure resilience division from the American Society of Civil Engineers (ASCE)–.

On the other hand, we believe that this research offers multiple novel research opportunities associated with analytical and computational modeling and optimization of realistic systems of interdependent networks. In particular, we consider that the following three themes are among the most critical and promising ones:

- Developing hierarchical approaches to study interdependent systems: models such as the td-INDP often focus on analyzing the systems at full resolution.

However, in some instances it may be a better idea to apply reduction techniques, in order to simplify the studied networks and facilitate the analyses and the implementation of strategies. By developing adequate hierarchical approaches for interdependent networks, the associated solution times may drastically reduce, and the modeling may better reflect realistic human-systems' dynamics. Hierarchical representation and hierarchical optimization approaches have been successfully used to model and analyze diverse infrastructure systems, such as power and transportation networks (Gómez et al., 2013; Alquthami et al., 2014; Anandalingam and Friesz, 1992; Kong et al., 2017; Costantino et al., 2013; Sun, 2014). However such approaches have focused on single isolated systems, and do not take into account features associated with interdependencies between multiple networks. Thus, the main associated challenge is to develop analytical and computational methodologies that enable analyzing and optimizing simplified versions of a full-resolution system of interdependent networks, while being able to quantify and guarantee the quality of the obtained solutions. A possible approach to this problem could use system identification techniques to determine the main dynamics present in the system, in order to reduce the number of analyzed elements and variables. In particular, methodologies such as Dynamic Mode Decomposition (DMD) could be use jointly with the proposed recovery operator, to determine the main recovery modes of the system and guide a dimensionality reduction process, while preserving as much information as possible regarding the full-resolution system.

- Developing diverse distributed and decentralized optimization techniques specific for interdependent networks: as described in Section 5.1, most of current models do not consider diverse relevant characteristics of realistic networks, specially when modeling the decision making process. There are few models that explicitly integrate optimization with decentralized decision making. On one hand, there are diverse agent-oriented computational approaches that could be studied, such as agent-based modeling and game-theory models with Bayesian updating. On the other hand, there are diverse analytical techniques that could be studied for the context of interdependent networks, such as distributed-control techniques, and decentralized gradient-descent methods, among others, which often offer optimality bounds and guarantees more readily. Moreover, whenever the socio-technical structure of the studied system resembles a hierarchical decision-making configuration, other modeling approaches such as multi-level optimization (Migdalas et al., 1998) would be suitable. In particular, a multi-level set up could be used to model decisions at multiple hierarchical levels associated with infrastructure systems, which could include decisions made by policy makers, followed by private and public operators and final users.

- Improving the proposed INDP models to be better suited for targeted-attacks analyses: Most of the presented INDP models were designed to study the recovery and resilience of systems subject to random or natural hazards. However, systems of interdependent networks such as critical infrastructure are constantly under targeted attacks. Thus, it would be ideal to extend the INDP models, to study the hazards as separate entities that are looking to reduce the system's performance. Approaches based on robust and stochastic optimization, as well as bi-, tri-, and multi-level optimization and interdiction problems may be useful to study such types of scenarios, for instance in the context of cyber-security and military applications (Bailey et al., 2006; Lim and Smith, 2007).

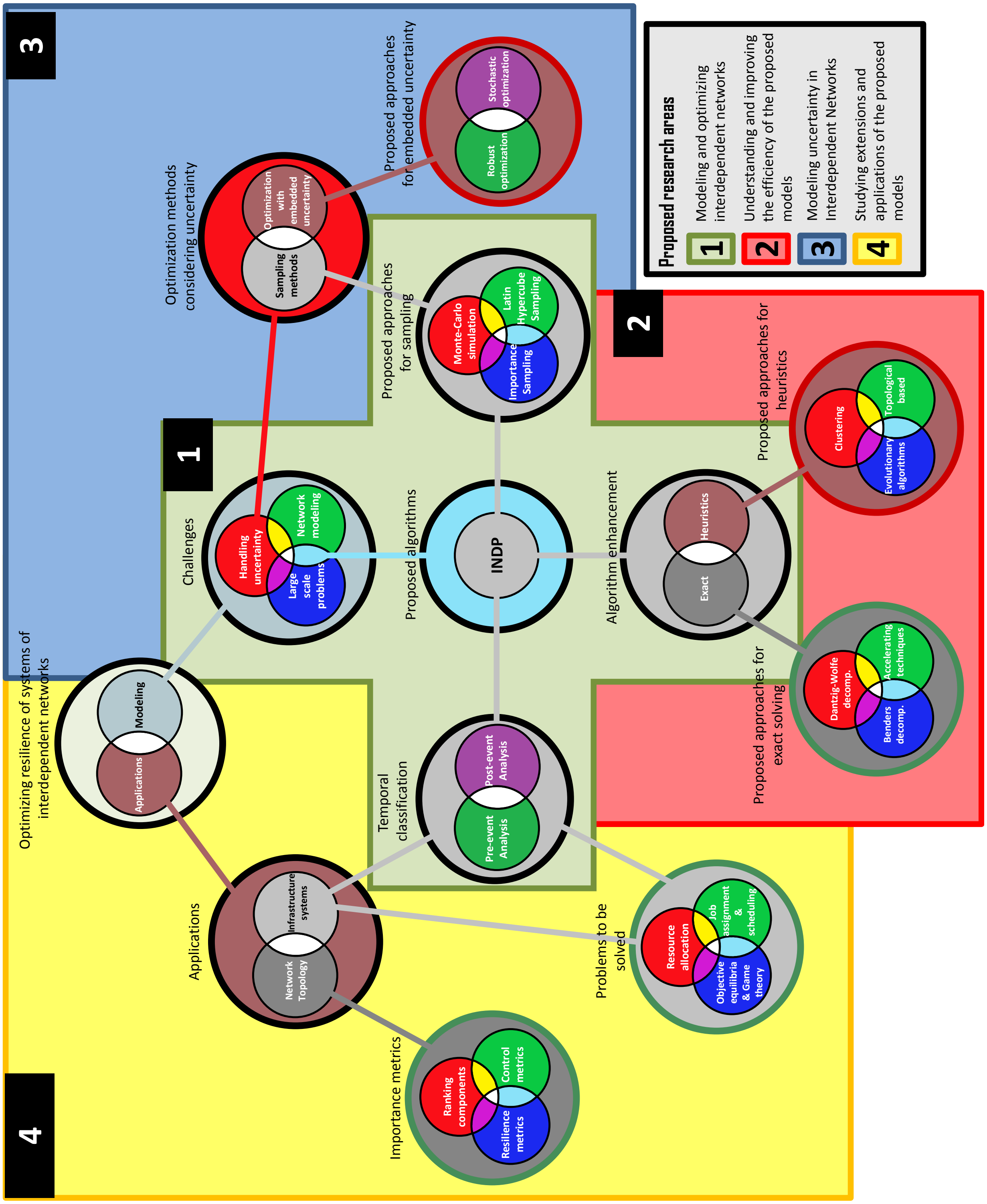


Figure 6.1 : Proposed research map divided by topics

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Appendices

Appendix A

List of Symbols & Abbreviations

A.1 Abbreviations

INDP	Interdependent Network Design Problem
tdINDP	Time-dependent Interdependent Network Design Problem
iINDP	Iterative Interdependent Network Design Problem
sINDP	Stochastic Interdependent Network Design Problem
NDP	Network Design Problem
TN	Tennessee
MIP	Mixed Integer Programming
MILP	Mixed Integer Linear Programming
PGV	Peak Ground Velocity
PGA	Peak Ground Acceleration
OR	Operations Research

A.2 INDP notation

A.2.1 td-INDP sets

$\mathcal{N}$	Set of nodes before a destructive event

$\mathcal{A}$	Set of arcs before a destructive event
$\mathcal{T}$	Set of periods for the recovery process (time horizon)
$\mathcal{S}$	Set of geographical spaces (spatial distribution of the area that contains the infrastructure networks)
$\mathcal{L}$	Set of commodities flowing in the system
$\mathcal{R}$	Set of limited resources to be used in the reconstruction process
$\mathcal{K}$	Set of infrastructure networks
$\mathcal{N}_k$	Set of nodes in network $k \in \mathcal{K}$ before a destructive event
$\mathcal{N}'_k$	Set of destroyed nodes in network $k \in \mathcal{K}$ after the event
$\mathcal{A}_k$	Set of arcs in network $k \in \mathcal{K}$ before a destructive event
$\mathcal{A}'_k$	Set of destroyed arcs in network $k \in \mathcal{K}$ after the event
$\mathcal{L}_k$	Set of commodities flowing in network $k \in \mathcal{K}$

A.2.2 td-INDP parameters

v_{rt}	Availability of resource r at time t
h_{ijkrt}	Usage of resource r related to recovering arc (i,j) in network k at time t
p_{ikrt}	Usage of resource r related to recovering node i in network k at time t
M^{+}_{iklt}	Costs of excess of supply of commodity l in node i in network k at time t
M^{-}_{iklt}	Costs of unsatisfied demand of commodity l in node i in network k at time t

α_{ijkst} Indicates if repairing arc (i, j) in network k at time t requires preparing space s

β_{ikst} Indicates if repairing node i in network k at time t requires preparing space s

$\gamma_{ijk\tilde{k}t}$ Indicates if at time t node i in network k depends on node j in network $\tilde{k}$

g_{st} Cost of preparing geographical space s at time t

f_{ijkt} Cost of recovering arc (i, j) in network k at time t

q_{ikt} Cost of recovering node i in network k at time t

c_{ijklt} Commodity l unitary flow cost through arc (i, j) in network k at time t

u_{ijkt} Total flow capacity of arc (i, j) in network k at time t

b_{iklt} Demand/supply of commodity l in node i in network k at time t

A.2.3 td-INDP decision variables

δ^{+}_{iklt} Excess of supply of commodity l in node i in network k at time t

δ^{-}_{iklt} Unmet demand of commodity l in node i in network k at time t

x_{ijklt} Flow of commodity l through arc (i, j) in network k at time t

w_{ikt} Binary variable that indicates if node i in network k is functional at time t

y_{ijkt} Binary variable that indicates if arc (i, j) in network k is functional at time t

Δw_{ikt} Binary variable that indicates if node i in network k is not damaged at time t

Δy_{ijkt} Binary variable that indicates if arc (i, j) in network k is not damaged at time t

Δz_{st} Binary variable that indicates if space s has to be prepared at time t

A.3 sINDP notation

A.3.1 sINDP sets

$\mathcal{N}$ Set of nodes before a destructive event

$\mathcal{A}$ Set of arcs before a destructive event

$\mathcal{T}$ Set of periods for the recovery process (time horizon)

$\mathcal{S}$ Set of geographical spaces (spatial distribution of the area that contains the infrastructure networks)

$\mathcal{L}$ Set of commodities flowing in the system

$\mathcal{R}$ Set of limited resources to be used in the reconstruction process

$\mathcal{K}$ Set of infrastructure networks

$\mathcal{N}_k$ Set of nodes in network $k \in \mathcal{K}$ before a destructive event

$\mathcal{N}'_k$ Set of destroyed nodes in network $k \in \mathcal{K}$ after the event

$\mathcal{A}_k$ Set of arcs in network $k \in \mathcal{K}$ before a destructive event

$\mathcal{A}'_k$ Set of destroyed arcs in network $k \in \mathcal{K}$ after the event

$\mathcal{L}_k$ Set of commodities flowing in network $k \in \mathcal{K}$

A.3.2 sINDP parameters

g_{st} Cost of preparing geographical space s at time t

f_{ijkt} Cost of recovering arc (i, j) in network k at time t

q_{ikt} Cost of recovering node i in network k at time t

u_{ijkt} Total flow capacity of arc (i, j) in network k at time t

α_{ijkst} Indicates if repairing arc (i, j) in network k at time t requires preparing space s

β_{ikst} Indicates if repairing node i in network k at time t requires preparing space s

$\gamma_{ijk\tilde{k}t}$ Indicates if at time t node i in network k depends on node j in network $\tilde{k} \in \mathcal{K}$

$c_{ijklt\omega}$ Commodity l unitary flow cost through arc (i, j) in network k at time t in scenario ω

$v_{rt\omega}$ Availability of resource r at time t in scenario ω

$h_{ijkrt\omega}$ Usage of resource r related to recovering arc (i, j) in network k at time t in scenario ω

$p_{ikrt\omega}$ Usage of resource r related to recovering node i in network k at time t in scenario ω

$M^{+}_{iklt\omega}$ Costs of excess of supply of commodity l in node i in network k at time t in scenario ω

$M^{-}_{iklt\omega}$ Costs of unsatisfied demand of commodity l in node i in network k at time t in scenario ω

$b_{iklt\omega}$ Demand/supply of commodity l in node i in network k at time t in scenario ω

P_ω Weight/probability of occurrence of each scenario ω

A.3.3 sINDP decision variables

Δw_{ikt} Binary variable that indicates if node i in network k is not damaged at time t

Δy_{ijkt} Binary variable that indicates if arc (i, j) in network k is not damaged at time t

Δz_{st} Binary variable that indicates if space s has to be prepared at time t

$\delta^+_{ikltω}$ Excess of supply of commodity l in node i in network k at time t in scenario ω

$\delta^-_{ikltω}$ Unmet demand of commodity l in node i in network k at time t in scenario ω

$x_{ijkltω}$ Flow of commodity l through arc (i, j) in network k at time t in scenario ω

$w_{iktω}$ Variable that indicates the functionality level of node i in network k at time t in scenario ω

$y_{ijktω}$ Variable that indicates the functionality level of arc (i, j) in network k at time t in scenario ω

Appendix B

Summary of publications & presentations developed during the Ph.D. studies

B.1 Journal papers

- González, A. D., Dueñas-Osorio, L., Sánchez-Silva, M., and Medaglia, A. L. (2016b). The Interdependent Network Design Problem for Optimal Infrastructure System Restoration. *Computer-Aided Civil and Infrastructure Engineering*, 31(5):334–350

We introduce the Interdependent Network Design Problem (INDP), which is the problem of defining the minimum-cost reconstruction strategy of a partially destroyed system of infrastructure networks, subject to budget, resources, and operational constraints, while considering interdependencies between the networks. Based on a Mixed Integer Programming (MIP) model of the INDP, we developed the algorithm denominated iterative INDP (iINDP) to optimize the full reconstruction process of a system of interdependent infrastructure networks after a disaster, while exploiting for the first time efficiencies and savings from joint restoration, due to co-location. In order to account for uncertainty related to possible disaster scenarios, we present a simulation framework that uses the iINDP to analyze the expected costs and performance associated to reconstructing the system of networks. To exemplify the capabilities of the presented methodology, we study the process of restoration of a set of interconnected *networks after a fictitious earthquake in Colombia and Shelby County, TN, USA. We analyze the evolution of the system performance and the reconstruction costs, providing an effective visualization tool for decision makers.*

- González, A. D., Chapman, A., Dueñas-Osorio, L., Mesbahi, M., and D'Souza, R. M. (2017a). Efficient Infrastructure Restoration Strategies using the Recovery Operator. *Computer-Aided Civil and Infrastructure Engineering*, (In review)

Infrastructure systems are critical for society's resilience, governments' operation and overall defense. Thereby, it is imperative to develop informative and computationally efficient analysis methods for infrastructure systems, while revealing their vulnerability and recoverability. To capture practical constraints in systems analyses, various different layers of complexity are required, such as limited element capacities, restoration resources, and the presence of interdependence among systems. High-fidelity modeling such as mixed integer programming and physics-based modeling can often be computationally expensive, making time-sensitive analyses challenging. Furthermore, the complexity of recovery solutions can reduce analysis transparency. An alternative, presented in this work, is a reduced-order representation, dubbed a recovery operator, of a high-fidelity time-dependent recovery model of a system of interdependent networks. The form of the operator is assumed to be a memory-less linear dynamic model. The recovery operator is generated by applying system identification techniques to numerous disaster and recovery scenarios. The proposed compact representation provides simple yet powerful information regarding the recovery dynamics, and enables generating fast suboptimal recovery policies in time-critical applications.

- Abolghasem, S., Gómez-Sarmiento, J., Medaglia, A. L., Sarmiento, O. L., González, A. D., Díaz del Castillo, A., Rozo-Casas, J. F., and Jacoby, E. (2017). A DEA-centric decision support system for evaluating Ciclovía-Recreativa programs in the Americas. *Socio-Economic Planning Sciences*, (In press)

Ciclovía-Recreativa (CR) is a community-based program with health and social benefits including physical activity promotion, social capital development, improvement in the population's quality of life, and reduction of air pollution and street noise. It is critical that these programs are evaluated through their operational performance and efficient use of resources. In this paper, we develop a DEA methodology that measures each CR efficiency relative to its peer programs, compares its performance to a benchmark system, identifies its sources of inefficiencies and offers recommendations for improvement. We examine the proposed methodology on programs in the region of the Americas as a case study and demonstrate the results and the recommendations. Finally, we present a spreadsheet-based DEA-centric Decision Support System (DSS) that facilitates the evaluation of the CR programs. Based on this study, an award called 'Bicis de Calidad' (in English 'Bikes of Quality') was created to be granted to the best CR programs reaching full efficiency according to the DEA outcomes.

- Smith, A., González, A. D., D'Souza, R. M., and Dueñas-Osorio, L. (2016). Interdependent Network Recovery Games. *Journal of Risk Analysis*, (In review)

Recovery of interdependent infrastructure networks in the presence of catastrophic failure is crucial to the economy and welfare of society. Recently, centralized methods have been developed to address optimal re- source allocation in post-disaster recovery scenarios of interdependent infrastructure systems that minimize total cost. In real-world systems, however, multiple independent, possibly noncooperative, utility network *controllers are responsible for making recovery decisions, resulting in suboptimal decentralized processes. With the goal of minimizing recovery cost, a best-case decentralized model allows controllers to develop a full recovery plan and negotiate until all parties are satisfied (an equilibrium is reached). Such a model is computationally intensive for planning and negotiating, and time is a resource that is crucial in post-disaster recovery scenarios. Furthermore, in this work, we prove that this best-case decentralized negotiation process could continue indefinitely. Accounting for the network controllers' urgency in repairing their system, we propose an ad-hoc sequential game theoretic model of interdependent infrastructure network recovery rep- resented as a discrete time noncooperative game between network controllers that is guaranteed to converge to an equilibrium. We further reduce the computation time needed to find a solution by applying a best response heuristic and prove bounds on ϵ-Nash equilibrium. We compare and contrast best-case and ad-hoc models on an empirical interdependent infrastructure network in the presence of simulated earthquakes to demonstrate the extent of the tradeoff between optimality and computational efficiency. Our model provides a foundation for modeling socio-technical systems in a way that mirrors restoration processes in practice.*

- González, A. D., Medaglia, A. L., Schaefer, A. J., Dueñas-Osorio, L. & Sánchez-Silva, M.(2017). The Stochastic Interdependent Network Design Problem. (Working paper)
- González, A. D., Medaglia, A. L., Sánchez-Silva, M. & Dueñas-Osorio, L. (2017). A hybrid algorithm to solve Interdependent Network Design and Recovery Problems.(Working paper)
- González, A. D., Sánchez-Silva, M., Medaglia, A. L., & Dueñas-Osorio, L.

(2017). Optimal resilience-based ranking for Interdependent Networked Systems. (Working paper)

- Chapman, A., González, A.D., Mesbahi, M., Dueñas-Osorio, L. & D'Souza, R., (2017). Understanding Restoration Dynamics and Controllability of Interdependent Networks using System Identification. (Working paper)

- González, A. D., Stanciulescu, I. & Dueñas-Osorio, L. (2017) Fractal based non-linear FEM analysis for trees bending under wind loading. (Working paper)

B.2 Peer-reviewed proceedings

- González, A. D., Dueñas-osorio, L., Sánchez-Silva, M., Medaglia, A. L., and Schaefer, A. J. (2017b). Optimizing the Resilience of Infrastructure Systems under Uncertainty using the Interdependent Network Design Problem. In *12th International Conference on Structural Safety & Reliability (ICOSSAR2017)*, pages 1–10, Vienna, Austria

Proper operation of critical infrastructure networks such as power, gas, water, and telecommunications is imperative for modern societies. Unfortunately, these systems are widely exposed to natural hazards, which combined with increased demands and high interdependency, increase their vulnerability at the community level exacerbating human and economic losses. In addition, interdependencies under disaster conditions increase the analytical and computational complexity of the problem if analysts want to probe systems ahead of time. In previous research, the Interdependent Network Design Problem (INDP) was proposed to optimize the recovery of a partially damaged system of in- terdependent networks, subject to resource and capacity constraints. However, recovery strategies depended on a specific disaster under study, preventing the INDP *from optimizing the systems before the occurrence of a given uncertain event. In this work, we introduce an approach to optimize systems both before and after the occurrence of a disaster, by expanding the capabilities of the INDP to incorporate uncertainty. This new approach simultaneously determines how to retrofit the system to enhance its robustness (before the occurrence of a disaster), and the best recovery strategies associated to a set of possible future damage sce- narios. Thus, the proposed model can effectively optimize the overall resilience of the analyzed system of systems. This is achieved by developing a two-stage optimization model, where in the first stage the optimal retrofitting strategy is found in preparation for the occurrence of a disastrous event, while in the second, the INDP formulation determines adequate recovery strategies associated to a set of possible disaster scenarios. To exemplify the capabilities of the pro- posed methodology, an illustrative study is carried out on the water, power, and gas networks of Shelby County, TN, subject to earthquake hazard given their proximity to the New Madrid Seismic Zone. Results show how the presented algorithm provides useful insights to decision makers to enhance the resilience of a realistic system of interdependent infrastructure networks under uncertainty, so that the impact of natural hazards is minimized.*

- Chapman, A., González, A. D., Mesbahi, M., Dueñas-Osorio, L., and D'Souza, R. M. (2017). Data-guided Control: Clustering , Graph Products, and Decentralized Control. In *56th IEEE Conference on Decision and Control (CDC2017)*, pages 1–8, Melbourne, Australia

This paper presents a novel approach to form an interdependent network model from time-varying system data. The research incorporates system meta-data using k-means clustering to form a layered structure within the dynamics. To compactly encode the layering, a Cartesian product model is fit to time-varying data using convex *optimization. In fact, we show that under special situations a closed form solution of this model can be acquired. The Cartesian form is particularly conducive to reasoning about the role of the interdependent network layers within the dynamics. This is illustrated through the derivation of a distributed LQR controller which requires only knowledge of local layers in the network to apply. To demonstrate the work's utility, the proposed methods and analysis is applied to time-series data from a high-fidelity interdependent infrastructure network simulation.*

- *González, A. D., Dueñas-Osorio, L., Medaglia, A. L., and Sánchez-Silva, M. (2016a). The time-dependent interdependent network design problem (td-INDP) and the evaluation of multi-system recovery strategies in polynomial time. In Huang, H., Li, J., Zhang, J., and Chen, J., editors, *The 6th Asian-Pacific Symposium on Structural Reliability and its Applications*, pages 544–550, Shanghai, China

Infrastructure systems are critical for societal resilience as well as government operation and defense. Thereby, it is imperative to perform thorough yet computationally efficient analyses on infrastructure systems, while quantifying their vulnerability and recoverability. Nevertheless, to be applicable in realistic scenarios, such analyses must consider different layers of complexity not commonly dealt with together, such as the existence of limited capacities and resources, and the presence of interdependence among different systems. This paper presents a mathematical formulation to solve to optimality the time-dependent Interdependent Network Design Problem (td-INDP), which seeks the minimum-cost recovery strategy of a partially destroyed system of infrastructure networks, subject to operational, resources, and budget constraints, while considering interdependencies between them. In addition, given a disaster scenario, it

*Wilson Tang Best Paper Award, APSSRA6 (2016) *is imperative to be able to evaluate in real time the feasibility and quality of pre-existing recovery plans. To assess this issue, this paper also presents a polynomial-time algorithm based on the td-INDP that determines the feasibility and optimal performance of a given recovery strategy. A realistic illustrative example shows the application of the presented td-INDP to optimally recover the interdependent system of water, power, and gas networks in Shelby County, TN, USA, subject to earthquake hazards due to its proximity to the New Madrid Seismic Zone (NMSZ). The results show the applicability of the presented td-INDP to realistic scenarios, offering a robust tool to support decision-makers in the design and evaluation of recovery strategies for multiple infrastructure systems at once.*

- González, A. D., Dueñas-Osorio, L., Sánchez-Silva, M., and Medaglia, A. L. (2015). The Computational Complexity of Probabilistic Interdependent Network Design Problems. In *Proceedings of the 12th International Conference on Applications of Statistics and Probability in Civil Engineering (ICASP12)*, pages 1–8, Vancouver, Canada

It is increasingly acknowledged that interdependence increases the vulnerability of infrastructure systems. Thus, it is imperative to consider the impact of infrastructure interdependencies, in particular when developing mitigation and recovery plans. González et al. (2014) recently introduced the Interdependent Network Design Problem (INDP), developed a Mixed Integer Programming (MIP) formulation to model and solve such a problem, and embedded it into a Monte Carlo simulation framework to account for uncertainties. The MIP and simulation framework allowed studying systems of interdependent infrastructure networks in order to provide relevant metrics quantifying the fragility and resilience of each of their components, and providing relevant information for mitigation planning and emergency response after disasters *strike. Nevertheless, the used MIP model is based on the Network Design Problem (NDP) which is an NP-complete problem, implying that the INDP model is at least equally computationally complex. Even more, given that the proposed framework requires the iterative use of the INDP model and numerical simulation, the computational complexity of such optimization model has a major impact on the overall time performance and problem size instances. Hence, we present a rigorous study of the computational complexity of the INDP solution model, detailing how each constraint in the formulation adds to the overall complexity and revealing how to tackle the computational demands for large interdependent networks. Realistic computational examples are presented, to illustrate the sensitivity of the INDP model to its parameters and constraints. Specifically, it is shown how some algorithms that are used to enhance the NDP, including decomposition techniques, can be adapted to the INDP model. In particular, tailor-made algorithms are presented to improve the time efficiency and handling of large size instances of the INDP model, and by extension, the overall simulation framework, enabling practical decision support and resilience analyses.*

- González, A. D., Sánchez-Silva, M., Dueñas-Osorio, L., and Medaglia, A. L. (2014b). Mitigation Strategies for Lifeline Systems Based on the Interdependent Network Design Problem. In Beer, M., Au, S.-K., and Hail, J. W., editors, *Vulnerability, Uncertainty, and Risk: Quantification, Mitigation, and Management*, pages 762–771. American Society of Civil Engineers (ASCE)

Critical infrastructure systems, such as water, gas, and electric power, are constantly stressed by aging and natural disasters among others. Between 2001 and 2011, adverse events such as earthquakes, landslides, and floods, accounted for economic losses exceeding USD 1.68 trillion, where around 30% is due to direct infrastructure damages. Hence, governments and other stakeholders must give priority to mitigate *the effect of natural disasters over such critical infrastructure systems, especially when considering their increasing vulnerability due to growing interconnectedness. The objective of this paper is to present a methodology that integrates Mixed Integer Programming (MIP) optimization methods into the calculation of metrics related to the reliability and resilience of a system of interdependent infrastructure networks, improving over mitigation strategies and vulnerability assessment that do not consider interdependence relations within an optimization framework. This methodology aims to provide tools to simulate diverse failure scenarios, assess their impact on the system's performance and recoverability, and quantify its sensitivity to adverse events. For the system's performance and recoverability measurement, this methodology builds upon the Interdependent Network Design Problem (INDP), which focuses on finding optimal recovery strategies given a disaster scenario. A computational experiment is carried out on simplified versions of the water, power, and gas networks in Shelby County, TN, which are geographically and physically interdependent. Results show that the approach permits comprehensive analysis about the fragility and vulnerability of a system of multiple correlated networks; hence, providing with useful information for decision makers to determine efficient mitigation actions directed to resilience enhancement.*

- González, A. D., Dueñas-Osorio, L., Medaglia, A. L., and Sánchez-Silva, M. (2014a). Resource allocation for infrastructure networks within the context of disaster management. In Deodatis, G., Ellingwood, B., and Frangopol, D., editors, *Safety, Reliability, Risk and Life-Cycle Performance of Structures and Infrastructures*, pages 639–646, New York, USA. CRC Press

Just in 2011, the economic losses associated to natural disasters worldwide accounted for more than USD 435 billion, which represents a significant 0.62% of the Gross World Product. Natural disasters often occur without enough warning and their *impact in the infrastructure networks is widespread and severe. Even more, given that infrastructure networks are ever increasingly interconnected, the problem of preventing, mitigating, and recovering from disasters across human and physical systems becomes a chronic difficult task. The objective of this paper is to present a new methodology to optimize resource allocation in infrastructure networks after a disaster, thus helping to prepare and recover systems of interconnected networks from a catastrophic event while acknowledging uncertainties and, for the first time, co-location interdependencies. The proposed methodology simulates disaster scenarios and their impact on a set of networks, and iteratively reconstructs them by evaluating the optimal recovery strategy. This strategy is developed subject to budget constraints and additional constructive and functional constraints embedded in a Mixed Integer Programming (MIP) optimization framework, denominated Interdependent Network Design Problem (INDP). This methodology also allows integrating the reconstruction sequence for different interconnected infrastructure networks simultaneously while exploiting efficiencies and savings from joint restoration. Hence, the proposed approach can be used to analyze, design and coordinate a repairing strategy for multiple geographically correlated utilities, instead of treating each system separately. The optimization-based mitigation actions and efficient responses after a disaster are summarized in evolution curves for the reconstruction, providing an effective visualization of the restoration processes and a new tool for decision makers in charge of multiple co-located infrastructure systems.*

B.3 Technical reports

- † Dueñas-Osorio, L., González, A. D., Shepherd, K., and Paredes-Toro, R. (2015). Performance and Restoration Goals across Interdependent Critical Infrastructure

†Chapter 8 of National Institute of Standards and Technology (NIST) (2016)

Systems. Technical report, National Institute of Standards and Technology (NIST), Houston, TX, USA

Infrastructure systems such as power, water, transportation, and telecommunication networks are vital for modern societies. They are essential for the safety, economic vitality, and overall governance and wellbeing of citizens. These systems continue to co-evolve, resulting in increased operational interdependencies and shared expectations of performance and restoration times. This co-evolution generally increases the reliability and efficiency of each individual system during normal operation, but the effects of interdependencies in the face of crises are not fully understood. Implementation of principles from the nascent field of interdependent lifeline systems have all increased dramatically over the last decade to help infrastructure owners, government agencies, and users understand how interdependencies manifest, as well as how they can manage them to support restoration efforts and fulfill community service expectations. However, societal expectations of performance are highly variable and context-dependent requiring empirical evidence as well as interdisciplinary modeling and analysis methods of lifeline system interdependencies. For this reason, scholars and practitioners are endeavoring to assess the gaps between desired multi-system performance from users, and expected performance from operators, so as to better support community resilience. This chapter provides initial trends on performance and restoration goals, along with taxonomies of modeling and simulation methods for the study of interdependent critical infrastructure systems and their restoration. We identify a need for research into the socio-technical aspects of infrastructure systems, and start building a tensor-like synthesis of restoration time gaps between utilities and societal functions or processes. Additionally, we assess promising areas for future research and development that will inform society and operators about performance and restoration times while supporting

the role of interdependent lifeline systems to community resilience.

B.4 Technical Talks

- González, A. D., Dueñas-Osorio, L., Medaglia, A.L., Sánchez-Silva, M., & Schaefer, A. (2016). *The Stochastic Interdependent Network Design Problem.* In Annual Meeting of the Institute for Operations Research and the Management Sciences (INFORMS). Nashville, TN, USA.

- González, A. D., Dueñas-Osorio, L., Medaglia, A. L., & Sánchez-Silva, M. (2016). *A hybrid algorithm to solve the time-dependent Interdependent Network Design Problem.* In Probabilistic Mechanics & Reliability Conference 2016 (PMC2016). Nashville, TN, USA.

- González, A. D., Dueñas-Osorio, L., Medaglia, A. L., & Sánchez-Silva, M. (2015). *Resilience Optimization as an Interdependent Network Design Problem.* In MPE 2013+ Workshop on Natural Disasters, from the Center for Discrete Mathematics & Theoretical Computer Science (DIMACS). Atlanta, GA, USA.

- González, A. D., Dueñas-Osorio, L., Sánchez-Silva, M., & Medaglia, A. L. (2015). *Recovery Dynamics based on the simultaneous use of System Identification and the Interdependent Network Design Problem.* In Workshop on Information Engines / Control of Interdependent Networks. Santa Fe, NM, USA.

- González, A. D., Medaglia, A. L., Dueñas-Osorio, L., & Sánchez-Silva, M. (2015). *Efficient Resilience Optimization of Interdependent Networks.* In Annual Meeting of the Institute for Operations Research and the Management Sciences (INFORMS). Philadelphia, PA, USA.

- González, A. D., Medaglia, A. L., Dueñas-Osorio, L., & Sánchez-Silva, M. (2014). *Improving the Computational Efficiency of the Interdependent Network Design Problem MIP Model.* In Annual Meeting of the Institute for Operations Research and the Management Sciences (INFORMS). San Francisco, CA, USA.

- González, A. D., Dueñas-Osorio, L., Medaglia, A. L., & Sánchez-Silva, M. (2014). *Enhanced component ranking based on the Interdependent Network Design Problem.* In Network Science (NetSci) conference. Berkeley, CA, USA.

- González, A. D., Medaglia, A. L., Dueñas-Osorio, L., Sánchez-Silva, M. & Gómez, Camilo (2013). *The effect of surrogate networks on the Interdependent Network Design Problem.* In Annual Meeting of the Institute for Operations Research and the Management Sciences (INFORMS). Minneapolis, MN, USA.

Appendix C

Failure and Recovery Database for a System of Interdependent Networks

Introduction

Considering the limited data available to perform recovery and resilience analysis on interdependent infrastructure networks, we have compiled relevant data to serve as a benchmark for such analyses. In particular, the current version of this database is developed using recovery strategies generated from the time-dependent Interdependent Network Design Problem (td-INDP) (González et al., 2016b,a). For future versions of the database, we are planning on considering uncertainty not only associated with the failure modes of the system, but also with the physical and logical properties of the studied system of infrastructure networks –for which we will use the Stochastic Interdependent Network Design Problem (sINDP)–. To generate this database, we used Xpress-MP 7.9 solver, in a computer with Windows 7 Enterprise, 32GB of RAM, and processor Intel Core i7-4770. The database is organized into three different folders. **Folder 1** contains data with failure probabilities for each component in the studied system, **folder 2** contains the recovery strategies generated using the td-INDP model, and **folder 3** contains the input data necessary to to run the td-INDP model if desired. The studied system of systems includes stylized versions of the water, gas, and power networks for Shelby County, TN, USA. Figure 2.3 shows the utility networks used.

The latest version of the full dataset can be found in `http://duenas-osorio.rice.edu/Content.aspx?id=2147483674`. The rest of this Appendix details the specific contents of each folder in the database.

C.1 Folder 1 - Failure probabilities

The probabilities of failure of each component (probMΞ.txt) are related to the magnitude of the disaster. File probMΞ.txt has the probability of failure of each component for the gas, power and water networks in Shelby County, TN, given an earthquake with magnitude $\Xi \in \{6, 7, 8, 9\}$. Below follows the list of parameters found in these files.

proba $\leftarrow$ Probability of failure of each arc. Format: *(starting node, ending node, network) probability*

probn $\leftarrow$ probability of failure of each node. Format: *(node, network) probability*

The numbering for the networks is 1 for water, 2 for gas, and 3 for power, for all the parameters described. The power and water networks were adapted from Hernandez-Fajardo and Dueñas-Osorio (2011, 2013), and the gas network from Song and Ok (2010). To calculate the failure probabilities, we also used the fragility curves and methods from Hazus (FEMA, 2013), and the works by Adachi and Ellingwood (2009, 2010)

C.2 Folder 2 - Failure and recovery scenarios

There are folders V$\mathbf{\Theta}$, where $\mathbf{\Theta} \in \{6, 12\}$ indicates the value of resources available used in the simulations. Inside each folder, the files are named following the format MΞV$\mathbf{\Theta}$T$\mathbf{\Psi}$RetCost200Iter$\mathbf{\Gamma}$.txt, where $\Xi \in \{6, 7, 8, 9\}$ indicates the simulated earthquake magnitude, $\mathbf{\Theta} \in \{6, 12\}$ indicates the available resources, $\mathbf{\Psi}$ indicates the maximum time horizon (calculated for each simulation separately), and $\mathbf{\Gamma} \in \{1, 2, ..., 1000\}$

indicates the label associated to each random simulation (of damage and associated restoration) performed (for each possible configuration of magnitudes and resources). Below follows the list of parameters found in these files.

M ← Moment magnitude

V ← Resources available (in this case, # of components that can be recovered per period)

TimeHorizon ← # of periods to perform the recovery process

Iteration ← Randomly generated disaster scenario (based on **M** and the fragility of each component)

ProblemStatus ← describes if the presented solution is the optimum

bestObjValue ← best objective function found (obtained by using the provided recovery strategy)

LB ← best lower bound found for the objective function (i.e., the optimal solution cannot be less that this value)

gap ← The percentage difference between the current solution and the best bound

time ← Time used by the optimizer to reach the provided solution

soltFOa ← Flow cost. Format: *(period) value*

soltFOb1 ← Construction cost associated to arcs. Format: *(period) value*

soltFOb2 ← Construction cost associated to nodes. Format: *(period) value*

soltFOb ← Total construction cost. Format: *(period) value*

soltFOc ← Shared Construction cost. Format: *(period) value*

soltFO ← Objective function without unbalance cost. Format: *(period) value*

soltFO2 ← Unbalance cost. Format: *(period) value*

soltFOT ← Total objective function. Format: *(period) value*

soltsumarcs ← Number of arcs recovered. Format: *(period) value*

soltsumnode ← Number of nodes recovered. Format: *(period) value*

soltss ← Number of components recovered. Format: *(period) value*

solunsDem ← Unsatisfied demand. Format: *(period) value*

a ← *functionality*= 0 indicates that the arc was destroyed in that earthquake simulation, 1 otherwise. Format: *(starting node, ending node, network) functionality*

n ← *functionality*= 0 indicates that the node was destroyed in that earthquake simulation, 1 otherwise. Format: *(node, network) functionality*

solx ← The flow of each commodity, through each arc, each time period. Format: *(starting node, ending node, network, commodity, period) flow*

soly ← *functionality*= 0 indicates that the arc is not functional, 1 otherwise. Format: *(starting node, ending node, network, period) functionality*

soldy ← *dfunctionality*= 1 indicates that the arc was repaired in that time period, 0 otherwise. Format: *(starting node, ending node, network, period) dfunctionality*

solw ← *functionality*= 0 indicates that the node is not functional, 1 otherwise. Format: *(node, network, period) functionality*

soldw ← *dfunctionality*= 1 indicates that the node was repaired in that time period, 0 otherwise. Format: *(node, network, period) dfunctionality*

soldp ← *excess* indicates the excess of commodity in each node, at each time period. Format: *(node, network, period) excess*

soldm ← *deficiency* indicates the deficiency of commodity in each node, at each time period. Format: *(node, network, period) deficiency*

C.3 Folder 3 - INDP Input data

In this folder there is a unique file name INDP_data.txt, which contains all the data associated to the costs, capacities, network structures, and others necessary to solve the td-INDP (except for the initial disaster scenario to be studied) using the formulation presented in González et al. (2016b,a). The parameters shown in this file do not have a time index, since for the recovery strategies presented in **folder 2** it was assumed that these parameters did not depend on the time period, i.e., they were constant through the recovery process.

Below follows the list of parameters found in this file.

v ← Resource availability. Format: *(resource) value*

c ← Unitary flow costs. Format: *(starting node, ending node, network, commodity) cost*

f ← Reconstruction cost of link. *Format: (starting node, ending node, network) cost*

q ← Reconstruction cost of node. Format: *(starting node, network) cost*

Mp ← Oversupply penalty. Format: *(starting node, network, commodity) cost*

Mm ← Undersupply penalty. Format: *(starting node, network, commodity) cost*

g ← Cost of space preparation. Format: *(space) cost*

b ← Demand. Format: *(node, network, commodity) demand*

u ← Link capacity. Format: *(starting node, ending node, network) capacity*

h ← Resource usage when reconstructing link. Format: *(starting node, ending node, network, resource) value*

p ← Resource usage when reconstructing node. Format: *(node, network, resource) value*

gamma ← Physical interdependence between components. Format: *(starting node, ending node, network 1, network 2) value*

alpha ← Nodes belonging to each space. Format: *(node, network, space) value*

beta ← Links belonging to each space. Format: *(starting node, ending node, network,space) value*

a ← Arcs in the system. Format: *(starting node, ending node, network) value*

n ← Nodes in the system. Format: *(node, network) value*